HMH Into Math®
California

Research Evidence Base

HMH research mission

HMH® is committed to developing evidence-based educational solutions, assessments, and professional services. To support this goal, the Efficacy Research Team collaborates with school districts and third-party research organizations to evaluate the impact of our programs and services on student outcomes, teacher practice, and school leadership.

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Introduction

Built upon the *California Mathematics Framework*, *Into Math California*® is a research-based comprehensive curriculum that centers on rich learning tasks. These tasks support student understanding of each module's Big Idea and the Content Connections and Drivers of Investigation that weave these ideas together.

Students build conceptual understanding and procedural skills through collaborative discussions. Teachers facilitate these discussions with comprehensive supports in every lesson, including Math Language Routines and guidance for identifying common errors and misconceptions.

With *Into Math California*, students develop a deep, long-lasting mastery of mathematics lesson by lesson and module by module. As students perform rich, hands-on tasks and engage in meaningful discourse, they grow in both their conceptual understanding and their strategic knowledge of math. Through engaging activities and practice, they will not only apply their newly acquired skills and knowledge but also continue to build their mathematical fluency.

Into Math California fuels rich, teacher-facilitated student discourse and hands-on learning. The result is deep understanding of mathematics and joy in learning. *Into Math California* ignites long-lasting mathematical mastery.

Built upon a foundation of mathematics education research and authored by leaders in the field, this document highlights the features of this cohesive, innovative program while explicitly demonstrating the research upon which it is based.

Purposeful, Connected Mathematics for All

"Math concepts and skills are important to all of school and life. Math provides a new way to see the world, the beauty of it, and the way you can solve problems that arise within it. However, math is much more: Math is critical thinking and problem solving, and high-quality math experiences also promote social, emotional, literacy, and general brain development" (Clements & Sarama, 2020, p. 2). A strong foundation in mathematics is essential for academic and life success. However, rather than serving as a gateway, mathematics often becomes a barrier to college and career advancement, with implications for economic growth and workforce readiness in STEM industries (Getz & Ortiz, 2016). Addressing disparities in STEM education, particularly for underrepresented groups, is crucial for expanding opportunities and meeting industry demands (Bland et al., 2024; IEEE Global Public Policy, 2023).

Mathematics programs that effectively support the development of essential 21st-century skills are structured by coherent learning progressions that build conceptual understandings and foster connections among areas of mathematical study and between mathematics and the real world (NCTM, 2014). The 2023 *California Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve* (*Mathematics Framework*), which serves to guide educators in helping all students fulfill the rigorous goals of the California Common Core State Standards for Mathematics (CA CCSSM), shares this position:

"The framework envisions mathematics in transitional kindergarten through grade five as a vibrant, interactive, student-centered endeavor of investigating and connecting the big ideas of mathematics. From transitional kindergarten through fifth grade, children experience enormous growth in maturity, reasoning, and conceptual understanding. They develop an understanding of such concepts as place value, arithmetic operations, fractions, geometric shapes and properties, and measurement. Students who have gained an understanding of math taught in the elementary grades and enter sixth grade viewing themselves as mathematically capable are positioned for success in middle school and beyond" (California Department of Education (CDE), 2024, Ch. 6, p.1).

HMH® Into Math California provides relevant, rigorous, real-world mathematics learning that fosters students' positive mathematics identities. The program is structured around big ideas and coherent learning progressions that utilize evidence-based pedagogy and practices to drive engaging, interactive mathematical investigations with real-world applicability extending well beyond the classroom.

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Relevant, Real-World Mathematics

The *California Mathematics Framework* emphasizes embedding mathematics in real-world contexts to boost the engagement and relevance of concepts and content. This emphasis has a well-established research base, as mathematics is inherently connected to the natural and physical world. To be deprived of appreciation for its beauty and practicality is to be denied the opportunity to flourish in one's life (Su, 2020). Decades ago, Piaget (1965) found that children driven by innate curiosity develop mathematical understanding through interactions with their environment. Humans are born with an intuitive and compelling sense of numeracy and pattern recognition (Devlin, 2006; Feigenson & Halberda, 2004). Effective curricula follow coherent learning progressions, linking mathematical concepts to real-world applications (NCTM, 2014). Rather than learning "facts" in isolation, connecting problem-solving tasks to students' experiences increases motivation (Alberti, 2013; Caine & Caine, 1991; Czerniak et al., 1999; David & Greene, 2007; Gaddy et al., 2014; Hammond, 2014; Kovalik & Olsen, 1994; Verschaffel et al., 2020). Contextual problems promote mathematical modeling (Fosnot & Dolk, 2001) and interdisciplinary learning (Russo et al., 2011).

Authentic problems, as defined by Özgün-Koca et al. (2019), have real purpose, relevance, and mathematical value, and they encourage discussion (CDE, 2024, Ch. 4). Cross et al. (2009) identified four key instructional elements in the development of conceptual understanding during early childhood: analysis, reasoning, creativity, integration, and real-world connections. Higher-order thinking strategies such as inquiry-based discussions and problem-solving help to develop critical thinking (Miri et al., 2007). Research continually underscores how active engagement—questioning, reasoning, explaining, and problem-solving—optimizes learning (Freeman et al., 2014; Gresalfi et al., 2009; Hiebert et al., 1996; Kapur, 2014; Martin, 2009; NCTM, 2014; Smith & Stein, 2018). The mathematization model involving real-world problem-solving improves mathematical literacy (Aguirre et al., 2024; Cai & Hwang, 2023).

Real-world connections are essential for students who have been historically underrepresented in STEM disciplines (Jackson et al., 2021; Young, 2014), as well as for advanced learners (Gainsberg, 2008; VanTassel-Baska & Brown, 2007). Culturally relevant pedagogy values students' knowledge and experiences and contextualizes learning with real-life applicability (Aguirre et al., 2013, 2024; Gay, 2018; Hammond, 2014; Ladson-Billings, 1995; Muñiz, 2020; Paris, 2012). Authentic performance tasks allow teachers to connect students' cultures and interests with mathematical learning (Lane, 2013). By using real-world data, teachers help students see mathematics as a meaningful tool for understanding and improving their communities (CDE, 2024, Ch. 2).

How *Into Math California* Delivers

Into Math California is designed to make mathematics learning real, relevant, and intellectually engaging for all students, inviting them to integrate their own funds of knowledge and deepen their understanding of how mathematics influences and enables possibilities in the world.

Engagement With the World of Math

As students begin each module, the World of Math engages them by linking mathematical concepts to history, culture, careers, and everyday life. Through teacher-led discussions, students use prior knowledge to explore the cultural and historical significance of math while completing activities that showcase contributions from diverse figures and cultures. By understanding the real-world applications and historical roots of math, students experience joy and "wow" moments and come to see themselves as mathematicians who can recognize their own identities within the curriculum.

Engagement With a Math Learning Community

Each World of Math activity takes about 15 minutes. Each topic's brief introduction is followed by a detailed teacher guide that provides probing questions and answers. A student-facing Turn & Talk question encourages learners to explore the meaning of math in the world around them and collaborate with classmates to devise a shared understanding of and appreciation for mathematics and its relevance in their lives.

Fostering Positive Mathematical Identity

Developing a positive and valued mathematics identity at a young age is essential to building a strong mathematical foundation. All students should be holistically empowered to participate meaningfully in mathematics learning. Effective, equitable mathematics instruction nurtures within children a positive mathematical identity and a strong sense of agency; it allows children to view themselves as capable, self-efficacious doers of math who construct their own mathematical understanding, while also experiencing firsthand the wonder, joy, and beauty of math (NCTM, 2020). Early math experiences predict later academic success, extending beyond classrooms to economic well-being and quality of life (Aguirre et al., 2024; Campbell et al., 2014; Clements & Sarama, 2016; Denton et al., 2002). Fostering a positive mathematics identity is crucial for establishing a strong mathematical foundation and fostering long-term achievement (Clements & Sarama, 2020), making it a central objective of mathematics education (Bishop, 2012; Larson, 2017).

Mathematical identity encompasses self-perceptions, emotions, and beliefs about math, influencing motivation, participation, and learning approaches. It is shaped by factors such as gender, race, culture, and socioeconomic status (Aguirre et al., 2013; Bishop, 2012; Froschl & Sprung, 2016). Developing persistence, agency, and self-efficacy is key to overcoming challenges (Bishop, 2012). "Two pillars support a positive math identity: the belief that you can do math and the belief that you belong" (Froschl & Sprung, 2016, p. 320). However, many students lack this self-perception and, as a result, exclude themselves from advanced mathematics (Darragh, 2015).

For students of color, academic identity intersects with racial and cultural identity (Nasir, 2002). Stereotype threat can harm students' attitudes and performance, but research shows that supportive classroom environments can mitigate these effects (Farrington et al., 2012; Walton & Cohen, 2011). Rivas-Drake et al. (2014) found that positive racial and ethnic identities boost academic engagement for African American, Latino, Asian American, and Pacific Islander students. Thus, fostering strong math identities is an equity issue (Aguirre et al., 2013).

Mathematics identity is dynamic, shaped by societal influences and classroom interactions. Power dynamics in group work can marginalize students and diminish confidence (Bishop, 2012; Langer-Osuna, 2016, 2017). To support math identity, Aguirre et al. (2013) recommend deep mathematical learning, valuing diverse knowledge, fostering community, and affirming students' identities. For English learners (ELs), Turner et al. (2013) advocate for validating their reasoning and encouraging participation. Engaging families and communities also bolsters students' academic and cultural identities (Ishimaru et al., 2015). Empowering all students in mathematics requires a holistic, equity-centered approach.

How *Into Math California* Delivers

Into Math California supports each student in developing a personal connection to mathematics, fostering a sense of capability and confidence as members of the math community and future STEM professionals.

Cultivating Mathematics Learning Communities

The program's instructional framework provides a dynamic learning model in which students operate collaboratively and effectively among whole-class sessions, small-group activities, and independent practice. *HMH Classcraft* provides ongoing opportunities for students to engage in mathematical discourse via Turn & Talks and group discussions. The program's games allow students to expand, apply, and challenge their learning while also having fun. Additionally, students go deeper and work together cooperatively on Module Projects. Mathematical Language Routines ensure that ELs participate fully in discursive activities and engage as contributing members of the classroom community.

Promoting Interest in STEM Learning and Careers

World of Math activities and Module Projects ignite students' interest in mathematics while helping them recognize its relevance to the real world and their own lived experiences. Students are encouraged to contribute their funds of knowledge to the classroom to enrich their understanding of mathematics and foster curiosity about learning. Building on this interest, resources that provide scaffolding and added challenges help each student succeed and feel a sense of belonging in STEM learning, both now and in the future.

Big Ideas in Mathematics

The *California Math Framework* advocates for a "big ideas" approach to mathematics instruction across all grade levels. A “big idea” in mathematics, as defined by Charles (2005), is a central concept that links multiple mathematical understandings into a coherent whole. Research strongly supports this approach, emphasizing its role in structuring knowledge, promoting transfer across topics and grade levels, and streamlining overcrowded curricula (Bruner, 1960; Wiggins & McTighe, 2005; Siemon et al., 2012; Nurnberger-Haag et al., 2024).

Within and across grade levels, a coherent math curriculum follows a logical sequence that reflects the hierarchical nature of mathematics (Schmidt et al., 2005). From elementary school onward, students must develop an understanding of big ideas that encompass both overarching mathematical concepts and essential reasoning processes (Cross et al., 2009). Effective programs build on children's intuitive mathematical thinking to help them solve problems and engage in reasoning (Carpenter et al., 2015). Foundational concepts such as number sense, geometry, and spatial measurement are particularly crucial for early learners (Cross et al., 2009).

A key component of structured mathematics learning is proficiency with essential topics (National Mathematics Advisory Panel, 2008). Learning progressions clarify instructional goals, improve classroom assessment, and help teachers determine where students are in their learning trajectory (Moss, 2022; Sáez et al., 2013). Studies show that secondary students who learn through conceptually driven, big-ideas-based curricula achieve more and enjoy math more (Boaler et al., 2018). Nurnberger-Haag et al. (2024) argue that this approach benefits both students and teachers by enhancing their content knowledge and instructional capacity.

The *California Math Framework* "encourages teachers to think about math as a series of big ideas that enfold clusters of standards and that connect concepts. And it encourages them to teach these ideas in multidimensional ways that meet the broad range of student learning needs" (CDE, 2024, Ch. 6, p. 2). Teaching big ideas is emphasized as a means of fostering equity and engagement by promoting inquiry-based learning experiences that spark curiosity and connect math to real-world contexts. The *Framework* also emphasizes Content Connections, calling for meaningful learning activities that integrate mathematical content and practices across grade levels and disciplines.

How *Into Math California* Delivers

Big Ideas: Vertical Alignment

Into Math California modules address the Big Ideas described in the *California Mathematics Framework*. The following table shows how the Big Ideas interconnect across grade bands (in this case, at the upper levels of the program).

Connecting Big Ideas

The Big Ideas connect to each other throughout *Into Math California*. As an example, the following diagram shows connections among the Big Ideas in Algebra I.

Focus, Coherence, and Rigor in Mathematics Learning

Student success in mathematics depends on a clear, structured learning path. Effective instruction must be intentionally designed to have a meaningful impact on student learning (Hattie et al., 2017). Mathematics programs that build essential 21st-century skills are structured through coherent learning progressions that connect mathematical concepts to real-world applications (NCTM, 2014). These progressions reflect the logical and hierarchical nature of mathematics (Schmidt et al., 2005) and help teachers effectively guide students through the curriculum (NCTM, 2014). Learning progressions are defined as carefully sequenced building blocks that students must master as they work toward higher mathematical understanding (Popham, 2008). Because mathematics learning is cumulative, instruction must emphasize coherence and connection rather than treating topics as discrete units (Fosnot & Jacob, 2010; Ma, 2010). Mathematics learning progressions support long-term student growth by clearly outlining curricular frameworks, instructional sequences, assessments, and levels of learning sophistication (Fonger et al., 2018).

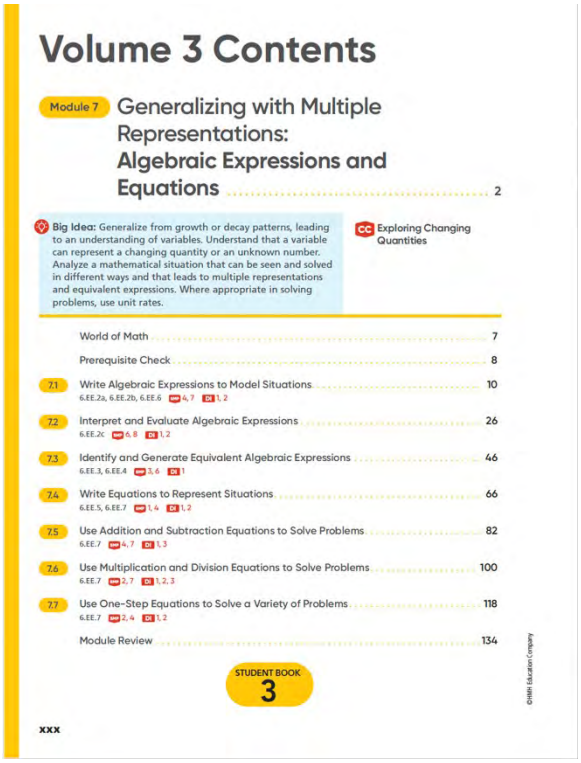
The *California Math Framework* promotes coherence by emphasizing big ideas, learning progressions across grades, real-world relevance, and high-quality instruction (CDE, 2024). A curriculum focusing on fewer but deeper mathematical concepts improves content mastery (Cobb & Jackson, 2011). The National Mathematics Advisory Panel (2008) recommends streamlining pre-K–8th-grade mathematics curricula to emphasize the most critical topics. Clearly identifying student learning expectations is crucial for effective instruction (Hattie et al., 2017; Wiggins & McTighe, 2005). Teachers must assess student progress and adjust instruction accordingly (Sarama et al., 2004; Sztajn et al., 2012). Instruction should also be intentional and responsive to students' learning needs, helping them understand the content and its purpose (Thompson et al., 2016; Wiliam, 2011). The *Framework* "provides focus by emphasizing the need for activities to target big ideas that necessitate understanding of multiple content and practice standards, emphasizing connections between topics that allow for deeper exploration, and use of tasks that are worthy of sustained student engagement" (CDE, 2024, Ch. 14, p. 7).

According to Hattie and colleagues (2017), rigor in mathematics education requires intentionally designed instruction that balances conceptual understanding, procedural fluency, and application, while providing clear success criteria. Effective programs build on students' intuitive thinking, supporting problem-solving and reasoning skills (Carpenter et al., 2015). For young learners, strong foundations in number sense, geometry, and spatial measurement are crucial for later mathematical success (Cross et al., 2009). "A robust curriculum is more than a collection of activities; instead, it is a coherent sequencing of core mathematical ideas that are well articulated across the grades" (NCTM, 2014, p.4).

How *Into Math California* Delivers

Purposeful, Connected, and Coherent Learning Progressions

While each lesson within *Into Math California* aligns with the state's rigorous CA CCSSM, program modules are built around Big Ideas. Lesson Planning pages clearly identify each of the specific California Common Core State Standards (CCSS), Big Ideas, Drivers of Investigation, Standards of Mathematical Practice, Content Connections, and learning objectives that will be covered. Standards are also identified throughout Student Edition lesson content, so learners recognize objectives and connections to Big Ideas. Such coherence helps teachers and students recognize the purpose and connections among content and activities, while maintaining focus on learning goals.



Volume 3 Contents

Module 7 Generalizing with Multiple Representations: Algebraic Expressions and Equations 2

Big Idea: Generalize from growth or decay patterns, leading to an understanding of variables. Understand that a variable can represent a changing quantity or an unknown number. Analyze a mathematical situation that can be seen and solved in different ways and that leads to multiple representations and equivalent expressions. Where appropriate in solving problems, use unit rates.

CC Exploring Changing Quantities

World of Math 7
Prerequisite Check 8
7.1 Write Algebraic Expressions to Model Situations 10
6.EE.2a, 6.EE.2b, 6.EE.6 MP 4, 7 DI 1, 2
7.2 Interpret and Evaluate Algebraic Expressions 26
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STUDENT BOOK 3

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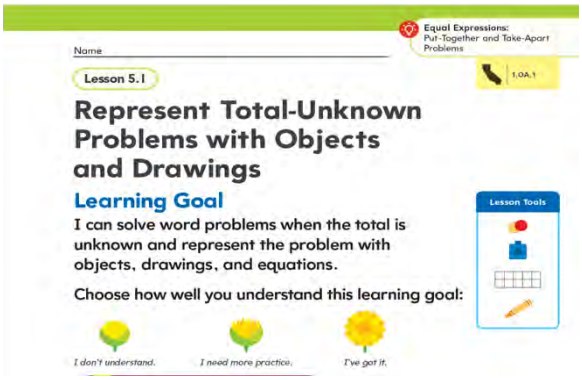
Learning Guided by Explicit Goals

Each *Into Math California* lesson makes learning goals clear and explicit to students, both at the beginning and as the lesson progresses. Lessons launch with a metacognitive awareness activity, asking students to reflect on how well they understand the goal. This enables students to recognize expectations and engage in purposeful, focused learning, while the program provides varied support to help them achieve their goals. *Into Math California* provides rigor with appropriate levels of challenge. All students can meet the *California Common Core State Standards for Mathematics* broadly, guided by clear and explicit learning goals in each math lesson.



Name

Equal Expressions: Put-Together and Take-Apart Problems

1.OA.1

Lesson 5.1

Represent Total-Unknown Problems with Objects and Drawings

Learning Goal

I can solve word problems when the total is unknown and represent the problem with objects, drawings, and equations.

Lesson Tools

Choose how well you understand this learning goal:

I don't understand. I need more practice. I've got it.

Drivers of Investigations and Connections

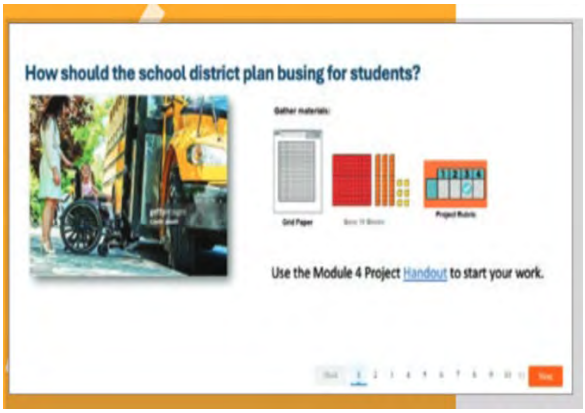
Mathematical investigations play a crucial role in engaging students and fostering deeper understanding by emphasizing inquiry-based learning, real-world problem-solving, and collaborative discourse. The *California Math Framework* advocates for instructional activities structured as investigations to spark curiosity, align with big mathematical ideas, and connect content with mathematical practices. Investigations follow a structured approach, guided by three key motivational drivers—sense-making, predicting, and impact—and are framed by the eight Standards for Mathematical Practice (SMPs). "[I]nstructional activities should be experienced as intriguing investigations designed to elicit questions about authentic, real-world contexts. Designed around the mathematical big ideas, these investigations are framed by a conception of the why, how, and what of math—a conception that makes connections across different aspects of content and also connects content with mathematical practices. Three Drivers of Investigation (DIs)—sense-making, predicting, and having an impact—provide the 'why' of an activity. They elicit curiosity and provide motivation. The eight SMPs provide the 'how.' Four types of Content Connections (CCs)—which ensure coherence throughout the grades—provide the 'what'" (CDE, 2024, Ch. 4, p. 6).

Student-centered investigations mirror the inquiry-based methods of mathematicians and scientists. These tasks are rich, open-ended, and involve activities such as observing patterns, conducting experiments, controlling variables, and constructing models. Discourse is central, with students collaboratively discussing problems, planning strategies, interpreting data, and evaluating solutions (Dorier & Maass, 2020; Langer-Osuna & Avalos, 2015; von Renesse & Ecke, 2015). Furthermore, effective investigations are designed to be inclusive, engaging diverse learners—including ELs and students with disabilities—by encouraging multiple solution approaches, representations, and collaborative problem-solving. An investigative and accompanying discursive approach to instruction cultivates students' positive mathematics identities (Langer-Osuna, 2016, 2017) and their mathematical habits of mind (Cuoco et al., 1996).

Research indicates that student-driven learning enhances engagement, enjoyment, and participation in mathematical discussions (Sengupta-Irving & Enyedy, 2015). However, shifting to investigative instruction requires teachers to transition from traditional methods to a facilitator role, guiding students through active learning and ensuring equitable participation (von Renesse & Ecke, 2015). Indeed, mathematical investigations have represented a gradual but significant shift over several decades, as documented by Sfard and Cobb (2014), from an acquisitive model to a participatory approach to math instruction. In this approach, teachers ideally identify learning goals and select real-world content or issues to frame learning activities. They anticipate collective learning trajectories based on students' individual assets, needs, backgrounds, and tendencies, and they carry out the activity in a facilitator role, creating a supportive classroom climate and ensuring each student contributes to and benefits equitably from the investigation. This shift may necessitate professional development, modeling, and scaffolding to support effective implementation (Yow & Lotter, 2016).

How *Into Math California* Delivers

Into Math California's Module Projects provide integral opportunities for collaborative problem-solving, creative thinking, and advanced mathematical reasoning through engaging, open-ended problems. Students explore the mathematics of each Big Idea while working at a Depth of Knowledge 4 level. These activities often use real-world scenarios and ask students to consider how math can have a genuine impact on the world (Driver of Investigation 3).



Igniting Curiosity, Promoting Investigation, and Forging Connection: Real-World Math

Curious students are engaged students. The Module Projects incorporate diverse experiences and topics, encouraging students to explore concepts more deeply. Each topic provides students with opportunities to connect the module's mathematics to their personal experiences and the experiences of others. The topics also encourage students to consider how math can be applied to make a positive impact on their community and the world.

Projects often address the themes of California's Environmental Principles and Concepts (EP&C). Many projects relate to environmental issues that students may experience locally or more broadly. By making connections between mathematics and the EP&C, students come to understand how math can help them care for the world around them.

Module 4 Project

Steering the Policy About Bus Service

Group Members: _____

How did you get to school? How do all students in the school? Some school districts have buses to take students to and from school every day. Some districts do not.

Think about working for a school district that has buses. Should every student be able to take the bus? You get to decide!

What is the project about?

In this project, you will write a busing policy for a school district by

- analyzing the data about students in the district,
- thinking about the available buses,
- using math to find the different options, and
- writing a policy that is supported with reason.

What are the steps of the project?

Check (✓) each step as you complete it.

- ☐ Review the rubric. Set a goal of how you want to rate your work.
- ☐ Answer these background questions with your group or for yourself:
  - How do you get to school?
  - Why are school buses important to schools and families?
  - What other times have you been on a school bus?
  - How do you think school districts decide who rides the bus?
- ☐ Now look at the details:
  - A rural school district with 428 students at 2 different schools.
  - Not all students live close to a school.
  - The district has 6 buses.
  - There is no budget for more buses.
  - Each bus can carry 3 students who use wheelchairs and 42 students who do not.
  - In this district, all students with special needs get busing to and from school.

Page 1

Optimizing Learning Experiences and Outcomes for All Students

Module Projects include teacher supports with suggestions for scaffolding each activity and student group organization to benefit all learning participants. Module Projects can also serve as optional performance assessments, each accompanied by a rubric to guide grading.

Mathematical Content and Practices: Exploration, Discovery, and Reasoning

"An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically" (NCTM, 2014, p. 7). The development of students' mathematical knowledge through effective teaching underpins strong mathematics instruction and profound understanding. Research demonstrates the importance of instructional tasks that cultivate conceptual understanding, facilitate discourse, and build procedural fluency, contributing to deep mathematical comprehension (NCTM, 2009 & 2014; NRC, 2012). High-achieving schools align curriculum, instruction, and assessment with standards, but standards alone are insufficient for real-world success (Peterson & Ackerman, 2015; Shannon & Bylsma, 2007; Tarr et al., 2008).

Mathematical literacy in the 21st century requires conceptual understanding (why), procedural fluency (how), application (when), and a strong math identity (Larson, 2017). Developing mathematical habits of mind is essential to mathematical proficiency, critical thought, college and career readiness, access to future opportunities, and productive participation in society (Goldenberg et al., 2015). Further, mathematical proficiency is developed via opportunities to engage in curiosity, investigation, argumentation, and collaboration around mathematical problem-solving. As the *California Mathematics Framework* emphasizes, math is an inherently social undertaking: "in many respects, mathematical knowledge and content understanding is developed and demonstrated socially; it is of little value to find a correct 'solution' to a problem without having the ability to communicate to others the validity and meaning of that solution. Thinking also can be clarified through exchange with others" (CDE, 2024, Ch. 4. p. 39).

HMH Into Math California empowers teachers with evidence-based tools, resources, and professional learning to improve outcomes and create an engaging classroom culture. The program is aligned to rigorous standards and promotes deep conceptual understanding and procedural fluency, number sense, problem-solving and reasoning, and data science competency by cultivating mathematical habits of mind, productive struggle, and discourse within a supportive learning community.

Standards for Mathematical Practices: Problem-Solving, Reasoning, Application, and Habits of Mind

The California Standards for Mathematical Practice (SMPs) emphasize problem-solving, abstract and quantitative reasoning, argument, precision, tool use, modeling, and critical thinking across all grades K through 12. The *California Mathematics Framework* aligns with SMPs by promoting inquiry-based learning through exploration, discovery, and reasoning (CDE, 2024). The goal is to prepare students to be confident problem-solvers who can apply mathematical thinking to real-world challenges. Proficient students actively engage in problem-solving, experiment with strategies, and persist through challenges, embodying the habits of mind essential for mathematical success (Cuoco, et al., 1996; Goldenberg et al., 2015).

Mathematical proficiency extends beyond memorizing formulas and procedures; it requires mastering cognitive strategies, perseverance, and reasoning skills. Drawing on the dynamic and discursive model of mathematical reasoning put forth by Jeannotte and Kieran (2017), The Framework encourages students to construct viable arguments, critique reasoning, recognize patterns, and express regularity in mathematical thinking. This approach aligns with decades of research advocating for mathematical habits of mind, emphasizing that true mathematical understanding stems from exploring and structuring problems rather than merely learning solutions (Cuoco et al., 1996). To foster these skills, students must regularly engage in challenging problem-solving tasks with multiple solution paths (Kaplinsky, 2019).

"[S]olving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim which is not immediately attainable" (Polya, 1965, p. ix). Problem-solving is an intrinsic part of human cognition, and within mathematics, it serves as a foundation for deeper conceptual understanding. Engaging students in formulating problems and assessing solution approaches enhances their ability to think critically and apply mathematical knowledge (Santos-Trigo, 2020), allowing them to actively construct understanding more deeply and meaningfully than when instruction is limited to teacher-led explanations and procedural exercises (Masingila et al., 2018). Research shows that student-centered problem-solving, rather than teacher-led procedural instruction, promotes longer-lasting comprehension and greater confidence in mathematics (Marcus & Fey, 2003).

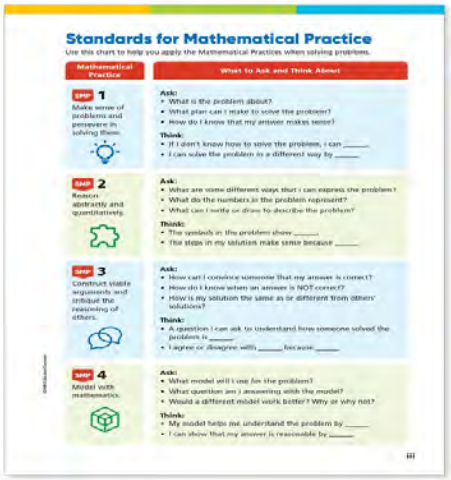
The *Framework* also stresses the importance of productive struggle, where students evaluate reasoning, justify answers, and refine their approaches based on feedback (CDE, 2024). Studies indicate that learning from failed problem-solving attempts before direct instruction results in greater conceptual understanding and knowledge transfer (Kapur, 2014). Effective mathematics instruction shifts responsibility to students, allowing them to assess the validity of solutions and engage in meaningful discourse (Mueller et al., 2014). By fostering problem-solving mindsets, teachers help students develop intellectual resilience and motivation, ultimately empowering them as lifelong mathematical thinkers (Blackwell et al., 2007).

How *Into Math California* Delivers

Into Math California closely aligns to and provides comprehensive support to aid students in meeting the rigorous CA CCSSM. Teachers and students find Drivers of Investigation, Standards for Mathematical Practice, and Content Connections clearly labeled and interwoven throughout the program, which also centers around Big Ideas.

Promoting Standards for Mathematical Practices

The program further supports students in developing SMPs. One way to achieve this is with the Standards for Mathematical Practice Anchor Charts.



To foster students' understanding of the SMPs, teachers can have students use the anchor charts in the front of the Student Edition as they work through the lessons. Teachers can also project or print copies to display in the classroom.

Application of Mathematical Processes to Reasoning and Problem-Solving and Cultivation of Habits of Mind

In addition to supporting all students in meeting rigorous standards, the SMP Anchor Charts serve to cultivate mathematical habits of mind, as they aid students in applying mathematical processes to reason and problem solve.

Into Math California Module Projects also provide consistent opportunities for students to collaboratively engage in open-ended problem solving, critical and creative thinking, and mathematical reasoning.

Number Sense

Having number sense entails deep understanding of numbers and their relationships. Number sense, which encompasses estimation, mental calculation, and number properties, is essential for mathematical fluency and flexible numerical thinking (Hiebert & Grouws, 2007; Baroody et al., 2012; McIntosh et al., 2005; Sood & Mackey, 2015). It underpins algebraic reasoning, including pattern recognition and equation manipulation (Chrysostomou et al., 2013), and serves as a predictor of future mathematics achievement (Sarama & Clements, 2009; Griffin, 2005; Jordan et al., 2010; Kirkland et al., 2024; Nelwan et al., 2022).

Number sense typically develops naturally in early childhood through environmental interactions such as sorting, comparing, counting and sequencing; number sense continues to develop into the secondary level and even adulthood (Clements & Sarama, 2007; Dehaene, 2011; Jordan et al., 2010; Mix et al., 2002; Piaget, 1965). Home and classroom experiences, including reasoning and play, further nurture numerical reasoning (Baroody et al., 2012; Sarama & Clements, 2009). Cognitive styles also shape students' problem-solving approaches and flexibility with numbers (Sarama & Clements, 2009).

The *California Mathematics Framework* (CDE, 2024) emphasizes that number sense develops throughout K–12 education. In early grades, students focus on counting, place value, and arithmetic. In Grades 3–5, flexibility with numbers extends to multiplication, division, and fractions (Boaler et al., 2018). Middle schoolers study rational numbers, proportional relationships, and number lines, while high schoolers explore polynomials, functions, and financial mathematics. Kirkland et al. (2024) found that mature number sense enables students to discern patterns and choose efficient problem-solving strategies—and it strongly predicts mathematics achievement.

Effective instruction requires hands-on experiences in early learning and explicit teaching starting in kindergarten (Baroody et al., 2012; Sood & Mackey, 2015). It is important to develop number sense alongside strategic reasoning (Sarama & Clements, 2009) and to provide frequent opportunities to build and apply number sense in real-world contexts, where numerical knowledge is ubiquitous (Griffin, 2005). Research shows that, across grade levels, games and math talks support the development of number sense by motivating students and providing practice and feedback (Baroody et al., 2012; Fuson et al., 2015; Griffin, 2005; Pan et al., 2022). Games featuring spaced pathways promote number sense among younger students at-risk for math difficulties (Ramani & Siegler, 2008). Well-crafted, frequent, and brief math talks foster students' mental math and computation skills (Boonen et al., 2011; Parrish & Dominick, 2014). However, the language teachers use, in general instruction and during math talks, is key. It must be accurate and avoid "tricks" or short-sighted rules. Rather than promoting memorized procedures, classroom discourse and instructional approaches should support students' authentic understanding of mathematical big ideas, including number sense (CDE, 2024; Hiebert & Grouws, 2007).

How *Into Math California* Delivers

In accordance with the CA CCSSM standards and research literature, *Into Math California* helps students develop number sense in early math learning and throughout every grade level. Number sense is embedded throughout program content.

Developing Number Sense via Play and Games

The program offers a variety of games at every grade level to support the development of mathematical proficiencies, including specifically number sense.

Developing Number Sense via Math Talk

The program fosters number sense by providing students with ongoing opportunities to engage in collaborative, supportive mathematical discourse with classmates. During these math talk activities, students are guided in recognizing and practicing numerical patterns.

Data Science: Mathematical Foundations and Reasoning with Data

Data literacy is an essential skill in today's data-driven world, influencing personal, professional, academic, and civic decision-making (Bargagliotti et al., 2020; Lee et al., 2021; Makar et al., 2023; NASEM, 2018; Smith et al., 2023; Wolff et al., 2016). For K–12 students, developing data literacy is crucial for engaging as informed global citizens. The study of data provides opportunities for authentic mathematical exploration, fostering conceptual understanding, reasoning, and problem-solving skills. Additionally, many careers require employees to interpret feedback data to refine practices, making data literacy a critical workforce skill. It also enables individuals to evaluate misleading information and make informed decisions regarding health, finances, and current events. "All students should develop the abilities not only to consume statistics and data with scrutiny, but also to think and work with data following a disciplined process of data inquiry" (Louie, 2022, p. 1).

The *California Mathematics Framework* emphasizes the importance of data literacy—also referred to as data science and data fluency—as a foundation for understanding various fields such as healthcare, economics, and politics (CDE, 2024). The framework highlights the growing career opportunities in data science, with job prospects expected to increase by 36% over the next decade (Bureau of Labor Statistics, 2022). Moreover, incorporating data science into K–12 education makes mathematics more engaging and relatable, fostering a sense of belonging for historically underrepresented groups in STEM, including women and students of color (Anwar et al., 2019; Chestnut et al., 2018; Jackson et al., 2021; Kang et al., 2019).

Wolff and colleagues (2016) define data literacy as the ability to inquire about and analyze real-world data, with a focus on ethical considerations. Critical data literacy extends this definition to include ethical data collection, privacy concerns, and power structures in data use (Bargagliotti et al., 2020; Louie, 2022; NASEM, 2018).

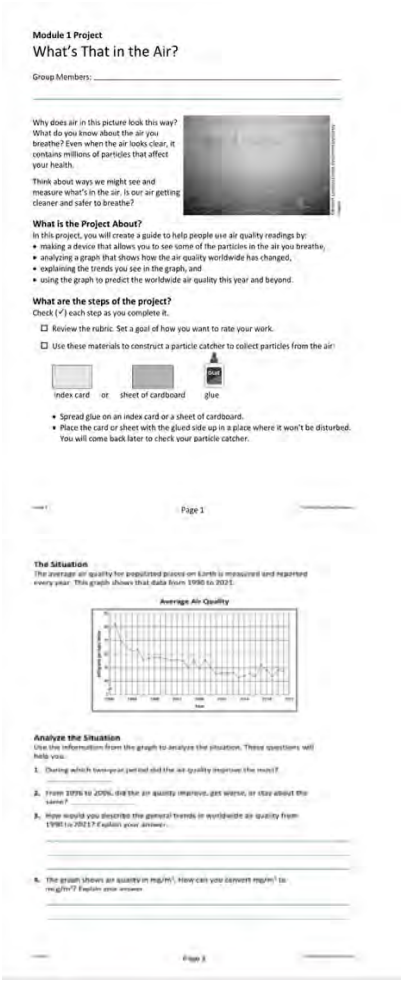
The pre-K–12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II) provides a framework for teaching data science, structured across three levels: elementary (basic data concepts), middle school (data variability and representation), and high school (sampling, probability, and inference) (Bargagliotti et al., 2020). The guidelines advocate for equitable access to high-quality statistics education, hands-on learning, dynamic visualization tools, and assessment methods that emphasize conceptual understanding over rote memorization. The *California Mathematics Framework* further supports embedding data science within SMP-based instruction, ensuring students gain essential skills in data collection, interpretation, and communication (CDE, 2024).

How *Into Math California* Delivers

Into Math California Module Projects provide ongoing opportunities for students to collect, display, and analyze data, as well as see the importance that such data plays in the world.

Data Literacy and Application in Real-World Math

Students working in print and digital Module Projects are guided step by step to achieve the activity's goals, which are centered around a series of probing questions that drive the investigation and make reporting more relevant and impactful.



Module 1 Project

What's That in the Air?

Group Members: _____

Why does air in this picture look this way? What do you know about the air you breathe? Even when the air looks clear, it contains millions of particles that affect your health.

Think about ways we might see and measure what's in the air. Is our air getting cleaner and safer to breathe?

What is the Project About?

In this project, you will create a guide to help people use air quality readings by:

- making a device that allows you to see some of the particles in the air you breathe,
- analyzing a graph that shows how the air quality worldwide has changed,
- explaining the trends you see in the graph, and
- using the graph to predict the worldwide air quality this year and beyond.

What are the steps of the project?

Check (✓) each step as you complete it.

☐ Review the rubric. Set a goal of how you want to rate your work.

☐ Use these materials to construct a particle catcher to collect particles from the air:

index card or sheet of cardboard glue

- Spread glue on an index card or a sheet of cardboard.
- Place the card or sheet with the glued side up in a place where it won't be disturbed. You will come back later to check your particle catcher.

Page 1

The Situation

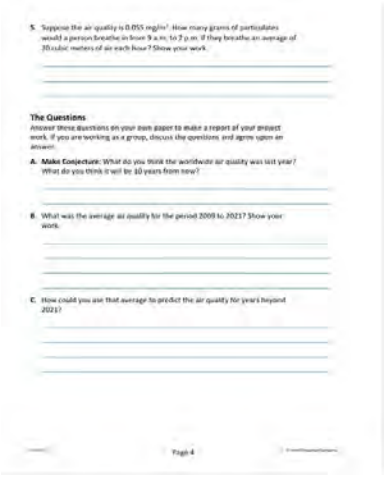
The average air quality for populated places on Earth is measured and reported every year. This graph shows that data from 1990 to 2021.

Average Air Quality

Analyze the Situation

Use the information from the graph to analyze the situation. These questions will help you:

1. During which two-year period did the air quality improve the most?
2. From 1996 to 2006, did the air quality improve, get worse, or stay about the same?
3. How would you describe the general trends in worldwide air quality from 1990 to 2021? Explain your answer.
4. The graph shows air quality in mg/m³. How can you convert mg/m³ to ng/m³? Explain your answer.



5. Suppose the air quality is 0.055 mg/m³. How many grams of particulates would a person breathe in from 9 a.m. to 7 p.m. if they breathe an average of 20 cubic meters of air each hour? Show your work.

The Questions

Answer these questions on your own paper to make a report of your project work. If you are working as a group, discuss the questions and agree upon an answer.

A. **Make Conjecture:** What do you think the worldwide air quality was last year? What do you think it will be 10 years from now?

B. What was the average air quality for the period 2009 to 2021? Show your work.

C. How could you use that average to predict the air quality for years beyond 2021?

Page 4

Developing Proficiency in Data Display and Communicating Ideas Visually

The digital portion of many Module Projects allows students to experiment with data displays to explore how different visualizations communicate ideas.

A partnership with Tuva Labs enables students to work with authentic data sets and tools throughout these activities.

Conceptual Understanding and Procedural Fluency: Teaching for Depth of Knowledge

U.S. education policy has long emphasized rigorous standards, aligned instruction, and accountability measures. While standards provide a foundation for effective teaching, they are insufficient on their own for improving learning outcomes. Focus and coherence are critical within standards-aligned instruction (Schmidt et al., 2005), particularly when adapting instruction to meet students' needs (Pak et al., 2020). Studies of mathematics instruction in the United States reveal a lack of depth and rigor, with content coverage often being overly broad (NRC, 2001). In contrast, high-performing countries in international comparisons focus on fewer topics taught with greater depth and coherence (NMAP, 2008; Schmidt et al., 2005). To address the coverage-versus-depth challenge, the *California Mathematics Framework* recommends focusing on Big Ideas, engaging tasks, and embedding exercises within larger investigations (CDE, 2024).

A longstanding debate in mathematics education centers on finding the right balance between conceptual understanding and procedural fluency. One side emphasizes exploratory, hands-on learning, while the other prioritizes rote practice. Research, however, underscores that both are essential and should develop in tandem through iterative learning (Baroody et al., 2009; Fuson, 2009; Larson & Kanold, 2016; NCTM, 2014; NRC, 2001; Rittle-Johnson, 2017). Procedural knowledge and conceptual understanding must be interwoven, as gains in one reinforce the other. Effective instruction fosters fluency with procedures while ensuring students understand the underlying concepts, allowing for flexible application of problem-solving skills and emphasizing meaning-making (Cross et al., 2009; Hattie et al., 2017; NRC, 2005; NCTM, 2014; Rittle-Johnson et al., 2015).

Conceptual understanding allows students to connect ideas, learn more efficiently, and assess the reasonableness of their solutions (NRC, 2001; Yang & Sianturi, 2019). Procedural fluency extends beyond memorization; it requires accuracy, efficiency, flexibility, and the ability to adapt strategies based on context (NRC, 2001, 2005). Developing fluency involves meaningful practice—engaging, purposeful, and spaced over time—rather than rote repetition (Baroody et al., 2009; Fuson & Murata, 2007; NCTM, 2015a; Rohrer, 2009).

To foster conceptual and procedural knowledge simultaneously, Rittle-Johnson (2017) suggests incorporating comparison (analyzing different problem-solving strategies), self-explanation (connecting new information to prior knowledge), and exploration prior to direct instruction. Students benefit from applying mathematics to real-world contexts, which reinforces both understanding and motivation (Alberti, 2013; Cross et al., 2009; David & Greene, 2007; Hiebert et al., 1996; NCTM, 2014).

How *Into Math California* Delivers

Built upon the *California Mathematics Framework*, *Into Math California* is a research-based, comprehensive curriculum centered on rich learning tasks that support students' understanding of each module's Big Idea, as well as the Content Connections and Drivers of Investigation that weave these ideas together.

High-Quality Instructional Materials and an Evidence-Based Instructional Framework That Build Conceptual Understanding and Procedural Fluency

Students build conceptual understanding and procedural skills through collaborative discussions. Teachers facilitate these discussions with comprehensive supports in every lesson, including Math Language Routines and guidance for identifying common errors and misconceptions.

The sequence of lessons follows instructional design that builds conceptual understanding, then connects concepts to skills, and ultimately allows students to apply their thinking. Open Middle tasks invite all students to engage in deep mathematical thinking.

Interactive Whole-Class Presentations With Real-Time Data Insights

Highly effective, interactive whole-class presentations provide teachers with curated resources to deliver engaging lessons supported by efficacious instructional routines.

Task-based Learning and Turn & Talk activities are facilitated via *HMH Classcraft*.

Deepening Mathematical Knowledge

Into Math California's multifaceted program components build deep mathematical learning. These include Module Projects, which offer rich, engaging Depth of Knowledge (DOK) Level 4 tasks in every module, allowing students to apply and extend their learning through interconnected opportunities for collaborative problem-solving, creativity, and advanced reasoning with open-ended problems. SMP Anchor Charts further support the development of mathematical habits of mind, encouraging students to engage in mathematical reasoning and problem-solving processes.

Effective, Timely, and Adaptative Practice for Personalized Learning

Into Math California provides opportunities to practice newly acquired and emerging skills across multiple modalities, including print, interactive digital platforms, teacher-led small groups, collaborative activities, and games.

The AI-powered *HMH Waggle*® platform allows students to practice math independently at the precise level most beneficial for their individual progression toward meeting the CA CCSSM.

Mathematical Discourse: Communicating and Collaborating as a Community of Learners

The *California Mathematics Framework* emphasizes the sociocultural foundations of effective mathematics learning, advocating for discursive, dialogic, and collaborative instructional approaches across all grade levels (Lambert & Sugita, 2016; Langer-Osuna & Avalos, 2015; Moschkovich, 2013; NASEM, 2018; NCTM, 2014; Smith & Stein, 2018; Turner et al., 2013). The *Framework* highlights that mathematical proficiency extends beyond individual cognitive ability, situating learning within social interactions. It underscores the integration of abstraction, generalization, and argumentation to support high-quality classroom discussions. "Teacher facilitation of high-quality mathematics discourse with attention to language development is the key to unlocking these practices for students and bringing them holistically into practice. Historically, proficiency in mathematics has been defined as an individual, cognitive construct. However, the past three decades of mathematics classroom research has revealed the ways in which learning and doing mathematics are rooted in social activity" (CDE, 2024, Ch. 4, p. 15).

Research has long demonstrated that verbalizing mathematical thinking enhances understanding and problem-solving abilities. Encouraging students to articulate problems before writing responses increases accuracy (Lovitt & Curtiss, 1968), while verbal expression combined with feedback fosters deeper learning (Gersten & Chard, 2001). “Communication is an essential part of mathematics and mathematics education…[that] can support students’ learning of new mathematical concepts as they act out a situation, draw, use objects, give verbal accounts and explanations, use diagrams, write, and use mathematical symbols…the communication process also helps build meaning and permanence for ideas and makes them public” (NCTM, 2000, p. 59–60). Furthermore, mathematical language development is key to removing learning barriers (Dacey et al., 2013).

Discourse-based mathematics instruction has been shown to enhance students’ reasoning and problem-solving across grade levels (Humphreys & Parker, 2015). Effective math instruction involves facilitating discussions in which students analyze and compare approaches, leading to greater engagement and confidence (Clements et al., 2023; Hufferd-Ackles et al., 2004 & 2015; Michaels et al., 2008; Morgan et al., 2014; Parrish & Dominick, 2016; Saylor & Walton, 2018). Encouraging students to justify and refine their mathematical thinking fosters deeper conceptual understanding (Carpenter et al., 2003). Additionally, discourse serves as a formative assessment tool, providing teachers insight into students’ thought processes (Franke et al., 2007; Hattie, 2009, 2012, 2023; Wiliam, 2018).

Math Talk has been identified as an equitable instructional practice, particularly benefiting ELs and historically marginalized students (Fuson & Leinwand, 2023; Langer-Osuna & Avalos, 2015; Turner et al., 2013). Quality discourse requires structured classroom protocols to promote respectful, collaborative exchanges. The *Framework* recommends principles such as valuing mistakes, fostering creativity, prioritizing depth over speed, and promoting equitable participation in mathematical discussions (CDE, 2024).

How *Into Math California* Delivers

Into Math California fuels rich, teacher-facilitated student discourse and hands-on learning. This approach fosters deep mathematical understanding and a joy of learning within supportive, inclusive communities, where language development is encouraged and every student contributes while maintaining a secure sense of belonging.

Structured and Supportive Classroom Learning Communities That Foster Language Development and Discourse

Students build conceptual understanding and procedural skills through collaborative discussions. For the benefit of all learners, including ELs, *Into Math California* features Math Language Routines and Activities, developed by Dr. Kate Kinsella and rooted in HMH's evidence-based *English 3D*®. Peer Coach videos model how teachers can effectively implement these routines, including detailed guidance on eliciting student responses. Mathematical Language Routines ensure that ELs participate fully in classroom discourse and engage as contributing members of the learning community.

Ongoing opportunities to share and co-construct understanding are provided through *HMH Classcraft's* Turn & Talk activities. *Into Math California*, in tandem with *HMH Classcraft*, provides continuous guidance to teachers for structuring learning activities and offers supports in every lesson, including strategies for identifying common errors and misconceptions.

SMP Anchor Charts support language development and classroom discourse by providing students with sentence and thought starters. Module Projects offer additional opportunities for collaborative and investigative work with classmates.

Dynamic Modalities Optimize Individual and Collaborative Differentiated Instruction

Into Math California enables data-driven, fluid instruction modalities that allow students to rotate seamlessly among whole-class sessions, small-group collaborations, and independent practice, supporting efficacious, differentiated, and dynamic instruction. Students have ongoing opportunities to work with teachers and peers, as well as independently, to develop conceptual understanding, procedural fluency, and a strong mathematical identity.

Mathematical Routines

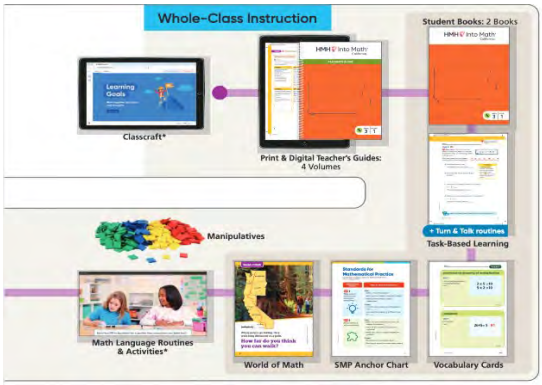
Mathematical proficiency requires deep thinking, which is best developed through structured, interactive instructional routines that engage students in reasoning, communication, and sense-making (Lampert, 2015; Berry, 2018). Predictable and repeatable classroom routines help students focus on mathematical tasks and on each other, fostering engagement and conceptual understanding. Well-designed reasoning routines allow students to articulate, revise, and refine their ideas both verbally and in writing (Zwiers, 2014). Establishing clear classroom norms ensures that students consider and build upon their peers' mathematical thinking, promoting collaborative learning (CDE, 2024, Ch. 2).

Effective instructional routines include “low floor/high ceiling” tasks, which provide accessible entry points for all students while scaling in complexity to challenge them at their individual levels (Sircar & Titus, 2015). These routines become second nature through repeated practice, much like classroom management routines (Kelemanik et al., 2016). Lampert (2015) suggests that structured routines should incorporate individual think time, peer discussions, public sharing of reasoning, and explicit connections to core mathematical concepts. Through repeated engagement in such routines, both teachers and students develop new intellectual and social skills that transform their perceptions of teaching and learning. Lucenta and Kelemanik (2020) emphasize a reasoning approach centered on mathematical modeling, guiding students through sense-making, analysis, model interpretation, adaptation, and reflection.

The *California Mathematics Framework* advocates for the use of structured mathematical routines to develop key proficiencies, particularly SMP.3, which involves constructing and communicating mathematical arguments. To support this, teachers should establish routines that enhance students' ability to express their ideas and engage in mathematical discourse within diverse classrooms (CDE, 2024, Ch. 4). One recommended routine is Claim, Evidence, and Reasoning (CER) (McNeil & Martin, 2011), which helps students justify conclusions using structured argumentation. The *Framework* also notes that well-chosen routines can support "math talks" that build SMP.1 and SMP.3. For example, Peer Revoicing (Cabana et al., 2014; Cohen & Lotan, 1997; Lagunoff et al., 2015; LaMar et al., 2020) “is a powerful routine for promoting shared understanding of mathematics as well as mutual recognition as young mathematicians. It structures the dialogue between the speaker and the listener in a way that ensures that the contributions build meaningfully upon each other. Teacher and peer revoicing can elevate the mathematical contributions of a student perceived as low status" (CDE, 2024, Ch. 2, p. 28). The *Framework* additionally recommends Zwiers and colleagues' (2017) Understanding Language (UL) Framework, which is discussed in greater detail elsewhere in this paper, with specific regards to supporting multilingual learners (MLs).

How *Into Math California* Delivers

Into Math California provides structured, task-based whole-group learning via *HMH Classcraft*. This program architecture aids teachers in instructional planning and classroom management and creates predictable, organized routines that support student learning.



Additionally, for the benefit of all learners, including ELs, *Into Math California* provides Math Language Routines and Activities, developed by Dr. Kate Kinsella and rooted in HMH's evidence-based *English 3D*®. Peer Coach videos are also available to model how to effectively carry out these routines.

Productive Struggle

Success in mathematics depends on practice and perseverance (Larson, 2016). "An effective teacher provides students with appropriate challenges, encourages perseverance in solving problems, and supports productive struggle in learning mathematics" (NCTM, 2014, p. 11). When students engage in productive perseverance, they grapple with problems, persist through difficulties, and build resilience, ultimately gaining confidence in their mathematical abilities (Jackson & Lambert, 2010).

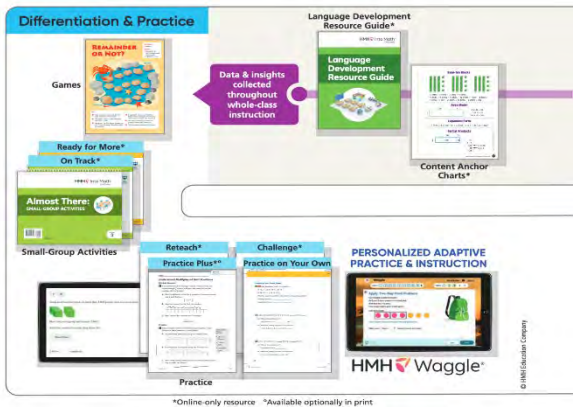
To develop mathematical habits of mind, educators must establish a classroom culture that normalizes challenge and emphasizes the benefits of perseverance (Star, 2015; Hiebert & Grouws, 2007). Growth mindsets—beliefs that effort and persistence lead to success—are essential for fostering productive struggle and improving academic achievement, particularly in mathematics (Dweck, 2006, 2008, 2015; NCTM, 2000, 2014). Learning contexts have a significant impact on students' capacities for establishing and sustaining growth mindsets (Yeager et al., 2022); teachers and schools that reinforce growth mindset messaging make learning enjoyable and place the focus on that learning rather than on students' performance (Yeager et al., 2013). A student's disposition toward mathematics significantly impacts their educational success (NRC, 2001). Research indicates that when students are allowed to make mistakes and persist through struggles, they demonstrate greater understanding (Hiebert & Grouws, 2007; Kapur, 2010, 2014; Warshauer, 2015). Perseverance in problem-solving also encourages metacognition, reinforcing the idea that meaningful learning comes from reasoning rather than simply arriving at correct answers (Hiebert & Grouws, 2007).

Teachers play a crucial role in fostering perseverance by carefully selecting tasks, providing guidance without lowering cognitive demand, and allowing sufficient opportunity for students to struggle productively (Dixon, 2018; Goldenberg et al., 2015). Effective questioning and timely scaffolding are key to helping students develop persistence rather than frustration (Warshauer, 2015). However, timing is critical: excessive upfront scaffolding can reduce cognitive demand and limit students' opportunities to engage in challenging mathematical thinking, raising equity concerns (Boston & Wilhelm, 2015; Dixon, 2018). Additional strategies for promoting perseverance include heterogeneous grouping, contextualizing problems in familiar settings, goal setting, reflection, and the use of "low floor/high ceiling" tasks (Sircar & Titus, 2015).

Productive perseverance supports a growth mindset, reinforcing that effort—not innate intelligence—determines success (Dweck, 2006, 2008, 2015). The *California Mathematics Framework* (CDE, 2024) emphasizes productive struggle as essential for deep understanding and problem-solving, and it urges teachers to challenge stereotypes about mathematical ability while fostering resilience in students.

How *Into Math California* Delivers

Into Math California promotes productive struggle by providing developmentally appropriate scaffolds that support each student in meeting the rigorous CA CCSSM without eliminating challenge.



Program scaffolding resources include Content Anchor Charts, the Language Development Resource Guide, manipulatives, Math Language Routines, games, and leveled small-group activities, which can be incorporated within each lesson's differentiated instructional framework and supported by *HMH Waggle*'s adaptive practice.

Teaching Mathematics for Equity and Engagement

"Improving mathematics access and outcomes in California requires that each classroom, transitional kindergarten through grade twelve (TK–12), is an equitable and engaging mathematics environment that supports all students" (CDE, 2024, Ch. 2, p. 1). Early experiences with mathematics yield effects well beyond classrooms, with consequences affecting economic prosperity, well-being, and quality of life (Campbell et al., 2014; Clements & Sarama, 2016). In today's technological society, where mathematics literacy is essential, limited access to effective mathematics instruction and its myriad applications can adversely impact an individual's economic, political, and civic engagement (Aguirre et al., 2024).

While mathematics achievement on every scale requires that all students be expected to meet rigorous standards, students come to school with unique backgrounds, skill sets, perspectives, strengths, and needs. Therefore, each student must receive effective, individualized support to realize and enjoy success in math learning (Aguirre et al., 2024; Clements & Sarama, 2020; Fuson et al., 2015; Hammond, 2014; Lawrence-Brown, 2004, NCTM, 2014, 2020; Shapka et al., 2006). All children, including those from historically underserved populations, are capable of learning and performing at high levels in mathematics, and a large body of research has documented the significant positive outcomes possible when schools and teachers address issues of equity and access (Gay, 2018; Gutiérrez, 2013; Kisker et al., 2012; Lawrence-Brown, 2004; Lipka et al., 2007; McKenzie et al., 2011). "Providing young children with extensive, high-quality early mathematics instruction can serve as a sound foundation for later learning in mathematics and contribute to addressing long-term systematic inequities in educational outcomes" (Cross et al., 2009, p. 2).

HMH Into Math California supports students equitably and effectively by providing access to highest quality mathematics instruction with embedded differentiation to meet high expectations and wide-ranging needs. The program also supports teachers by providing tools to help create nurturing, inclusive, and adaptive classroom environments that facilitate deep learning of mathematics for all.

Teaching for Equity in Mathematics

For over two decades, the National Council of Teachers of Mathematics (NCTM) has emphasized equity in mathematics education, advocating for inclusive and high-quality instruction that supports all students (NCTM, 2000, 2014). "An excellent mathematics program requires that all students have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential" (NCTM, 2014, p. 59). Despite growing demands for a STEM-trained workforce, historically marginalized groups—including female, African American, Latinx, and Native American students, as well as ELs, students in poverty, and those with disabilities—continue to face systemic barriers in mathematics education (Anwar et al., 2019; Jackson et al., 2021; Kang et al., 2019; Sneider & Ravel, 2021). Resolving these disparities requires recognizing that achievement gaps stem from unequal access to resources and opportunities, not from inherent cultural or socioeconomic differences (Aguirre et al., 2013; Flores, 2007; Gutiérrez, 2013; Lawrence-Brown, 2004; NCTM, 2014, 2022; Ukpokodu, 2011).

Equitable mathematics instruction prioritizes fostering positive mathematical identities, encouraging student engagement, and ensuring all students have access to rigorous learning experiences (Larson, 2018; NCTM, 2014, 2020. The *California Mathematics Framework* (CDE, 2024) reinforces this equitable approach by promoting culturally responsive teaching, open-ended problem-solving, and instructional strategies that validate students' backgrounds and lived experiences. Research underscores the importance of an asset-based approach in which students' cultural and linguistic identities are viewed as strengths rather than deficits (Aguirre et al., 2013, 2024; Gay, 2018; Hammond, 2014; National Council of Supervisors of Mathematics (NCSM) & TODOS, 2016). This perspective, also known as "funds of knowledge" (Moll et al., 1992), encourages teachers to intentionally build upon students' unique backgrounds by connecting new instructional content to their existing knowledge. It also emphasizes maintaining high expectations for all students, providing differentiated instruction, and ensuring inclusive participation norms (Aguirre et al., 2013, 2024; Gay, 2018; Hammond, 2014; Ladson-Billings, 1995, 2014; Muñiz, 2020; Paris, 2012). "[M]any of the critical challenges facing racial and ethnic minority students in the formation of strong, positive mindsets for academic achievement can be alleviated through the careful work of creating supportive contexts that provide consistent and unambiguous messages about minority students' belonging, capability, and value in classrooms and schools" (Farrington et al., 2012, p. 34).

Universal Design for Learning (UDL) frameworks promote multiple means of engagement, representation, and assessment, allowing students to demonstrate mathematical understanding in varied ways (Strangman et al., 2004). Equitable access also includes digital tools, ensuring all students—not only those in advanced courses—engage with high-cognitive-demand tasks through technology (White et al., 2021). Ultimately, advancing equity in mathematics hinges on shifting traditional instructional models to foster success for all students rather than reinforce barriers (NCTM, 2014).

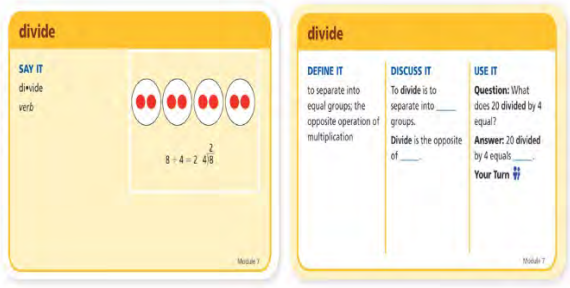
How *Into Math California* Delivers

Into Math California promotes conceptual understanding through student-driven discourse and hands-on learning. Students use manipulatives throughout lessons to deepen their mathematical understanding. The lesson sequence follows an instructional design that first builds conceptual understanding, then connects concepts to skills, and finally allows students to apply their thinking.

Ensuring Equitable and Effective Mathematics Learning

Universal Design for Learning (UDL) principles, resources for multilingual learners, and accessibility supports address the needs of all learners. Furthermore, the program views students' backgrounds, experiences, and prior math knowledge as funds of knowledge—valuable assets that are leveraged daily to ensure personal and cultural relevance for each student.

Into Math California provides highly effective, interactive, whole-class presentations enhanced by real-time data insights with Classcraft. Classcraft provides curated resources for engaging lessons supported by efficacious instructional routines. Resources that promote academic language development are embedded throughout *Into Math California*, including multilingual learner support, vocabulary cards, Dr. Kate Kinsella's language-building routines, and language development resource guides. Dr. Kinsella also helped create the program's Words to Learn routine, integrated into the vocabulary cards in every student book, to help students master essential math terms. These cards guide students to SAY, DEFINE, DISCUSS, and USE new words. Teachers can use printed copies of the cards for games and word walls, and the module planner provides a list of key terms along with blank templates for further customization.



Additionally, every teacher and student component of *Into Math California* is available in Spanish.

Ongoing, Seamless Differentiation and Practice Opportunities

Into Math California pinpoints student learning needs using detailed data and reporting tools available through *HMH Ed*®. Teachers can access targeted resources to differentiate instruction for students who need additional support, are on track, or require greater challenge. The program's instructional model enables differentiation before practice, incorporating formative assessment early in the learning process so teachers can address misconceptions with small groups before they become ingrained.

In addition to the practice built into each lesson, *Into Math California* includes:

- Practice Videos that showcase the first problem in Practice on Your Own (Practice on Your Own is included in every lesson.)
- Practice Plus Workbook for every grade with Spiral Review
- Personalized Adaptive Practice via Waggle
- Almost There Small-Group Mini Lessons, Reteach, Challenge, and Fluency

Into Math California also provides highly effective, interactive whole-class presentations with real-time data insights through Classcraft. Classcraft provides curated resources for engaging lessons supported by efficacious instructional routines.

Teaching for Engagement in Mathematics

"Students benefit from viewing mathematics as a vibrant, interconnected, beautiful, relevant, and creative set of ideas. As educators create opportunities for students to engage with and thrive in mathematics and value the different ways questions and problems can be approached and learned, many more students view themselves as belonging to the mathematics community" (CDE, 2024, Ch. 2 p. 7). To optimize engaged learning, the *California Math Framework* promotes inquiry-based learning, real-world applications, and diverse representations—such as visuals, movement, and technology—to enhance students' understanding and enjoyment of math.

Decades of extensive research have consistently shown that, across all education levels, content areas, and socioeconomic factors, active engagement in classroom activities is essential for meaningful and successful learning (Eccles et al., 1998; Frommelt et al., 2021; Jansen et al., 2022; Lee & Shute, 2010; Schunk & Mullen, 2012; Wang & Degol, 2014). Engagement encompasses behavioral, emotional, and cognitive dimensions (Fredricks et al., 2004), and motivation plays a crucial role in academic success (Ainley, 2012; Bandura, 1986; Ryan & Deci, 2000; Schunk et al., 2008). Active participation fosters persistence through challenges and strengthens knowledge retention (Chi & Wylie, 2014; Finn & Zimmer, 2012). Yet, research indicates that student motivation and engagement tend to decline in secondary education, undermining long-term academic success and participation in STEM fields, particularly among historically disadvantaged students (Duckworth et al., 2007; Kuo et al., 2021; Morgan et al., 2014).

In mathematics education, research supports approaches that connect math to real-life applications, increasing relevance and engagement (Aguirre et al., 2024; Cai & Hwang, 2023; Schoenfeld, 2002). Inquiry-based learning encourages students to ask questions, explore multiple pathways to solutions, and cultivate intellectual curiosity (Cabana et al., 2014; Deslauriers et al., 2019; Louie, 2017; Sengupta-Irving & Enyedy, 2015). Open-ended, problem-based tasks provide opportunities for students with diverse learning styles, linguistic backgrounds, and experiences to demonstrate their understanding in various ways and engage through varied modes of participation (CAST, 2018; Lambert & Sugita, 2016; Wong et al., 2003). Collaborative reasoning and group problem-solving further enhance engagement and mathematical understanding, as peer discussions promote conceptual clarity and spark interest (Langer-Osuna, 2017; Webb et al., 2014). Boaler and Sengupta-Irving (2016) found that students identified collaboration as the most impactful factor in boosting their engagement and interest in mathematics.

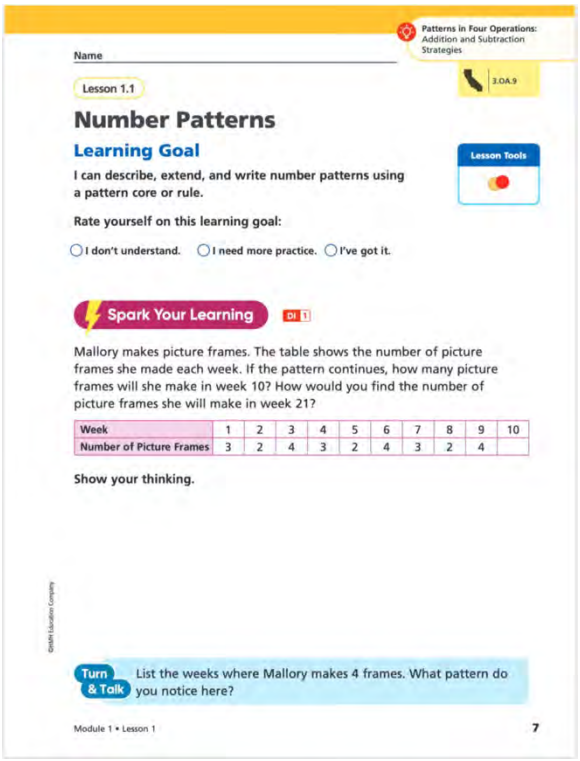
Using multidimensional teaching strategies—such as visuals, manipulatives, and technology—aligns with UDL principles, making math more accessible to all students (CAST, 2018). Engaging instructional practices also foster positive math identities, helping students see themselves as capable mathematicians and supporting their long-term success (Walton et al., 2012; Langer-Osuna, 2017).

How *Into Math California* Delivers

Into Math California infuses engagement with mathematics learning across all elements and learning arcs within the program.

Igniting Interest and Engagement in Learning

Lessons begin by identifying learning goals and engaging students in Get Ready activities, which generate self-assessment related to those goals. Spark Your Learning activities then capture students' interest and ignite curiosity. Turn & Talks engage students in interactive, collaborative peer learning, supporting the co-construction of understanding.



Patterns in Four Operations: Addition and Subtraction Strategies

Name

Lesson 1.1

3.OA.9

Number Patterns

Learning Goal

I can describe, extend, and write number patterns using a pattern core or rule.

Lesson Tools

Rate yourself on this learning goal:

○ I don't understand. ○ I need more practice. ○ I've got it.

Spark Your Learning DI 1

Mallory makes picture frames. The table shows the number of picture frames she made each week. If the pattern continues, how many picture frames will she make in week 10? How would you find the number of picture frames she will make in week 21?

Week	1	2	3	4	5	6	7	8	9	10
Number of Picture Frames	3	2	4	3	2	4	3	2	4	

Show your thinking.

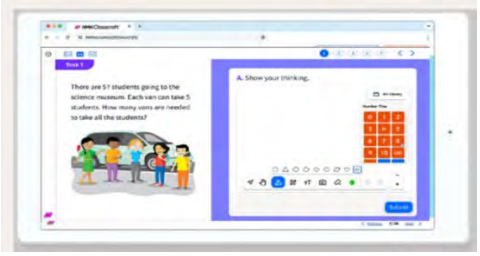
Turn & Talk List the weeks where Mallory makes 4 frames. What pattern do you notice here?

Module 1 • Lesson 1 7

Making Learning Interactive and Dynamic

Other program features further engage students in math learning:

- World of Math helps students see math learning as relevant, intriguing, and interconnected.
- Dynamic classroom structure and lesson frameworks stimulate and re-engage interest and interactivity across the span of a lesson as students shift from whole-class or small-group learning to independent practice rotations.
- Module projects provide hands-on, interactive engagement in interdisciplinary, real-world math learning.
- Dynamic digital learning, such as *HMH Ed*, *Classcraft*, and *Waggle*, offer multimedia appeal and interactivity. The *HMH Classcraft* Student Portal provides interactive learning experiences that helps students stay on task during whole-group, partner, and individual work. Within the same classroom, students can follow along using their book or digital resources—or a combination of both. *HMH Waggle* delivers engaging, adaptive practice, providing customized learning pathways to success. <<image from G4 implementation guide p26>>



Culturally Responsive Teaching and Culturally Sustaining Pedagogy

A culturally responsive pedagogy simultaneously attends to students' academic success, cultural competence, and social-political consciousness; it is both student- and community-driven (Ladson-Billings, 2014). The *California Mathematics Framework* emphasizes culturally responsive teaching and culturally sustaining pedagogy as essential for equity and engagement in math education. This approach acknowledges students' diverse backgrounds, experiences, and cultural perspectives, aiming to create meaningful and accessible learning experiences while maintaining high expectations for all learners (CDE, 2024, Ch. 1). NCTM (2015) underscores the importance of ensuring access and equity by addressing systemic factors that contribute to disparities in mathematics achievement across racial, ethnic, linguistic, gender, and socioeconomic groups.

Culturally responsive teaching, rooted in Ladson-Billings' (1995) foundational work, seeks to bridge gaps between school and home cultures by valuing students' lived experiences as assets to learning (Gay, 2018; González et al., 2006; Hammond, 2014; Paris, 2012; Wolfe, 2022). Research indicates that ineffective teaching practices and exclusionary policies contribute to the underperformance of minority and low-income students, highlighting the need for systemic reform (Gutiérrez, 2013; LaMar et al., 2020; Muñiz, 2020; NCSM & TODOS, 2016; Ukpokodu, 2011). Aguirre et al. (2013) advocate for a holistic equity framework that acknowledges the cultural identities of children of color and non-majority backgrounds, situating their learning within the local and broader social contexts in which they live.

To guide educators in implementing culturally responsive teaching, Muñiz (2020) outlines eight key competencies: reflecting on one's cultural lens, addressing systemic bias, integrating students' cultural backgrounds into instruction, incorporating real-world issues, setting high expectations, promoting respect for diversity, engaging families and communities, and ensuring linguistically and culturally responsive communication. These strategies align with research emphasizing the importance of participatory and collaborative learning, such as math talks and inquiry-based problem-solving, which foster student agency and engagement (Aguirre et al., 2013; Turner et al., 2013).

Equitable mathematics instruction requires rigorous expectations alongside support that addresses students' cognitive, emotional, and cultural needs (Gay, 2018; Hammond, 2014; Ladson-Billings, 2014; LaMar et al., 2020; Lawrence-Brown, 2004). When classrooms are structured as inclusive learning communities, students develop leadership in learning, engage in critical discussions, and recognize the impact of privilege and power in educational settings (Kozleski, 2010). By embedding culturally responsive and sustaining pedagogies, educators can foster a more inclusive mathematics learning environment that empowers all students.

How *Into Math California* Delivers

The *California Mathematics Framework* is brought to life through student-centered, collaborative learning. Each lesson begins with a Spark to activate prior knowledge and address misconceptions. Students then engage in high-quality, discourse-rich tasks that build conceptual understanding. Features like World of Math and Module Projects support the *Framework*'s Drivers of Investigation, encouraging students to reason, model, and collaborate as they make sense of the world.

Curious, Confident Problem-Solving and Culturally Relevant, Real-World Learning

With *Into Math California*, students see themselves as curious and confident problem solvers who embrace mistakes and demonstrate persistence. They also recognize the role math plays in the world and in their own lives.

Spark Your Learning Activities: Each Lesson begins with a "Spark Your Learning" activity designed to help students bring their own ideas and prior knowledge to the math concept.

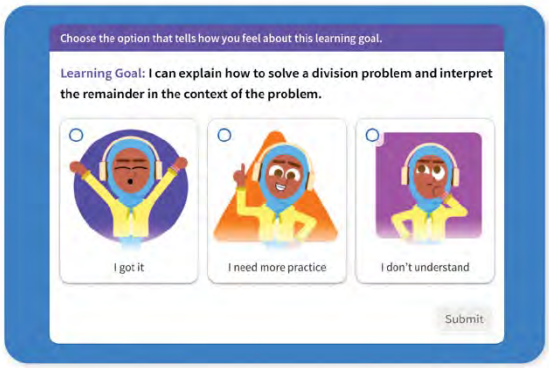
Collaborative discussions: Students build conceptual understanding and procedural skills through collaborative discussions. Teachers are supported in every lesson with comprehensive guidance, including math language routines and guidance for identifying common errors.

World of Math: The World of Math helps students understand that mathematics is more than just numbers and formulas. It has a rich and diverse history and influences every student's daily life. Students explore historical figures who contributed to mathematics and examine contributions from multiple cultures.

Module Projects: Rich and engaging Depth of Knowledge (DOK) Level 4 projects are provided in every module for students to apply and extend their learning. These projects make math relevant and relatable through real-world connections, problem-solving, and critical analysis of data.

Promoting Positive Math Identities and Student Agency

Learning Goal routines assess students' confidence with a learning goal at the start of each session. This not only promotes student ownership and relatability with learning goals but also enables teachers to tailor instruction to meet the needs of all learners.



Universal Design for Learning

Full, active, and contributory participation within the instructional environment is essential for equitable education (LaMar et al., 2020; Lambert & Sugita, 2016; Louie, 2017; Moschkovich, 2013) and positive math identities (Aguirre et al., 2013; Boaler et al., 2000; Chestnut et al., 2018; Wolfe, 2022).. The *California Mathematics Framework* integrates UDL principles (CAST, 2018) to promote accessibility and equity in math education, benefiting students with disabilities, English learners, and advanced learners. "UDL is a particularly useful framework because it does not separate students with disabilities and assume that their fundamental needs and/or learning processes are different from those of students without disabilities. Instead, UDL is focused on learner variability" (Lambert, 2020, p. 4). Within the *Framework*, UDL provides a proactive, multidimensional design approach for curriculum and instruction, ensuring that learning opportunities are adaptable to the diverse needs of all students. Further, UDL in the *Framework* aims to remove systemic barriers and foster equity by developing students' motivation, knowledge, and strategic thinking (LaMar et al., 2020). This approach aligns with the Common Core State Standards for Mathematics (CCSS-M) to ensure all students achieve high levels of mathematical proficiency (Lambert, 2020).

Lambert and Sugita (2016) propose that limited engagement with mathematical problem-solving curriculum is a primary reason why students with learning disabilities (LD) are underrepresented in STEM fields at the undergraduate level. In alignment with the research-based UDL guidelines developed and advocated for by CAST (2018), which support educators and stakeholders in establishing optimal learning conditions for all students, the *Framework* emphasizes three primary principles underpinning inclusive and equitable mathematics instruction.

First, multiple means of engagement (affective networks and the "why" of learning) provide students with varied ways to engage with mathematical content based on their interests, cultural backgrounds, and learning preferences. This includes options for recruiting interest, sustaining effort and persistence, and supporting self-regulation. Second, multiple means of representation (recognition networks and the "what" of learning) offers diverse ways for students to access and process information, such as through visual aids, manipulatives, graphs, and real-world contexts. This includes options for perception, language and symbols, and comprehension. Finally, multiple means of action and expression (strategic networks and the "how" of learning) allows students multiple ways to demonstrate their understanding, such as through written work, oral presentations, or hands-on projects. This includes options for physical action, expression and communication, and executive functions.

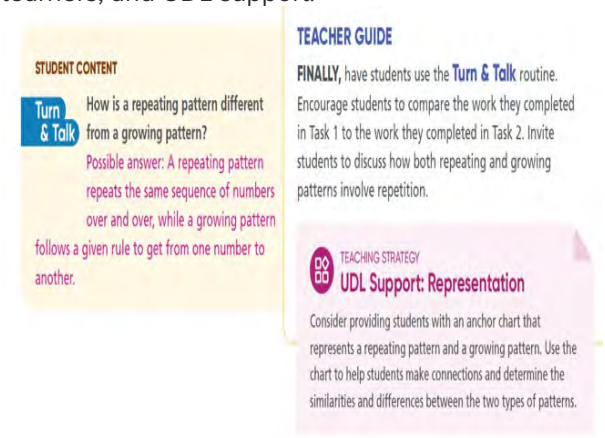
Creating an inclusive classroom environment includes offering accessible resources, prompting students to explain their reasoning, and involving paraprofessionals to support student learning (Lambert & Sugita, 2016; Baxter et al., 2005). Fostering inclusive discussions supports equitable education for all students (Foote & Lambert, 2011; Louie, 2017).

How *Into Math California* Delivers

Universal Design for Learning (UDL) principles, resources for multilingual learners, and accessibility supports are embedded in *Into Math California* to meet the needs of all learners. Students' backgrounds, experiences, and prior math knowledge are leveraged to ensure personal and cultural relevance for each student. This equitable, inclusive math experience allows every student to thrive, regardless of their background.

Inclusive and Supportive Learning Environments

Dr. Kate Kinsella's Key Routines in every module and grade level ensure multilingual learners feel comfortable in their classroom environment. The Turn & Talk activities that are included in every lesson foster a supportive learning environment. Students participate in meaningful conversations with partners, making their thinking visible to both the teacher and classmates. Point-of-Use scaffolds are embedded in every lesson, and targeted interventions support students who need additional help in mastering key concepts. *HMH Classcraft* also includes embedded supports for common errors, multilingual learners, and UDL support.



Accessible Learning Activities With Targeted Support

The Practice on Your Own feature allows students to practice what they have learned and includes Robert Kaplinsky's Open Middle® problems. These low-floor/high-ceiling activities are accessible to all students, encourage advanced problem-solving, and promote deep exploration and reasoning.

Students and parents can watch engaging, step-by-step videos demonstrating how to solve practice problems. These videos not only make learning enjoyable but also ensure students can grasp the concepts.

Almost There small-group activities and tabletop flipchart mini-lessons are perfect for co-teachers or resource math teachers. Detailed instructions, suggested questions, and additional materials support small-group instruction. The lessons enhance problem-solving and math talk.

Multilingual Support: Dual Content and Language Development

As of the 2023–24 academic year, nearly 20% of California's K–12 public school enrollment was comprised of multilingual learners (MLs). Spanish remains the predominant home language, spoken by over 869,000 MLs, followed by Mandarin, Vietnamese, Arabic, and Russian (CDE, 2025. In California, as in other parts of the United States, "multilingual learners come from a wide range of cultural, linguistic, educational, and socioeconomic backgrounds and have many physical, social, emotional, experiential, and/or cognitive differences. All bring assets, potential, and resources to schools that educators must leverage to increase equity in standards-based systems. Increasing avenues of access, agency, and equity for all multilingual learners" (WIDA, 2020, p. 18).

Researchers increasingly reject the deficit-based model of English instruction, which marginalizes MLs by focusing solely on what they lack—English proficiency—rather than recognizing their multilingual assets (August & Shanahan, 2007; Cummins, 2021; García, 2009; Hamman-Ortiz & Prasad, 2025). Evidence supports dual-language instruction, which develops foundational skills in students' heritage language (L1) and promotes skill transfer to English (L2), enhancing metacognition and academic success (Baker et al., 2016; Escamilla et al., 2014; Lindholm-Leary & Block, 2010). This approach leverages Common Underlying Proficiency (Cummins, 1991) and provides comprehensible input that boosts identity and cognitive growth (Krashen, 1996; Delgado, 2010). Drawing on theories and practice of dual language learning, translanguaging is an evidence- and assets-based approach that allows MLs to utilize their rich repertoires of linguistic and background knowledge to construct meaning and build skills both in English and across academic content areas. Translanguaging invites students to switch between L1 and L2 to build connections between the two and bridge home and school experiences. For MLs, this approach has been shown to support learning, empower effective communication, strengthen academic development, and increase opportunities for engagement and collaboration (Creese & Blackledge, 2015; Cummins, 2021; Flores & Schissel, 2014; García, 2009; Hamman-Ortiz & Prasad, 2025; Li, 2022; Seltzer, 2019; Vallejo & Dooly, 2020; Wei, 2018). Research-based best practices for MLs recommend clustered placement within grade-level content learning along with dedicated time for English learning and practice. Within both settings, intentional grouping plays a critical role, as it allows flexible opportunities to provide targeted support, peer interaction, confidence building, and translanguaging (Kinsella, 2016). To develop academic and discursive proficiency, MLs need to communicate daily and actively with peers of diverse English proficiency levels within structured, supportive environments where roles and goals are clear (August & Shanahan, 2007; Foster & Ohta, 2005; Kinsella, 2016; Saunders & Goldenberg, 2010).

Supporting MLs in math is crucial due to the subject's language demands across grade levels, particularly as instruction places greater emphasis on word problems and real-world application (Janzen, 2008). These demands contribute to difficulties in comprehension and assessment for MLs (Alt et al., 2014; Arizmendi et al., 2021). Research shows that dual language approaches enhance MLs' math performance, boost literacy skills (Bibler, 2021; Cartwright et al., 2022; Morita-Mullaney et al., 2021; King & Powell, 2023), and foster confidence, identity, and executive functioning (Castellón et al., 2024). Kinsella (2013) emphasizes the critical role of explicit, targeted vocabulary instruction to support MLs in content area learning, as it demands knowledge of high-utility and domain-

specific vocabulary to comprehend and respond effectively. However, because it is impossible to teach every unfamiliar word, Kinsella instead advocates for a strategic focus on words essential to understanding key concepts and facilitating academic discourse.

Research also demonstrates that consistent instructional routines are essential for enabling students to focus on content and language acquisition (Feldman & Kinsella, 2005;; Zwiers et al., 2017). Well-designed routines for reasoning provide opportunities for students to articulate complex mathematical situations, revise and refine their ideas, and strengthen their verbal and written expression (Zwiers, 2014). Through these routines, students engage collaboratively and creatively in mathematical sense-making (Lucenta & Kelemanik, 2020). For MLs, such routines can structure explicit instruction in vocabulary, syntax, and grammar, as well as authentic practice in language usage (Dutro & Kinsella, 2010; Kinsella, 2016).

How *Into Math California* Delivers

HMH knows that a student's ability to communicate a mathematical idea is closely related to the student's mastery of mathematical skills and concepts. *Into Math California* is carefully designed to provide teachers with opportunities to help students develop their command of mathematical language and content simultaneously. All students benefit from the program's language routines and discourse activities. Peer Coach Videos model how routines are effectively carried out in classrooms.

Kate Kinsella's Key Routines

Dr. Kate Kinsella, author of HMH's *English 3D*, brings her powerful key routines to *Into Math California*. These routines appear at the beginning of each module to help students develop academic language and achieve success throughout the module's tasks, as well as in their classroom conversations and interactions.

Observable, Measurable Language Objectives Within Each Lesson

Each *Into Math California* lesson includes Language Objectives that teachers can use to observe and measure the academic language growth of students. Each objective offers an appropriate language skill that is closely aligned with the lesson's content and *California English Language Development Standards*.

Teaching Strategies That Support Mathematical Communication

The Teacher's Guide includes specific Teaching Strategies that help you support your students as they work through each lesson's Language Objective. Each Teaching Strategy provides tools that you can use to help your students demonstrate the skills described in the Language Objective. Many strategies offer different levels of scaffolding, allowing students to choose the level of support that best meets their individual learning needs.

Words to Learn Routine and Vocabulary Cards

The Words to Learn (WTL) instructional routine, which can be found in *HMH Classcraft*, is designed to engage students in guided interactive word study, aid student comprehension of key academic vocabulary, and help students improve their understanding of high-utility words. The WTL routine works in tandem with the Vocabulary Cards for each module and are found toward the back of each Student Edition. The Vocabulary Cards complement the WTL routine, but they can also be incorporated into classroom activities, such as vocabulary games or word walls. Key vocabulary is also available at the start of each lesson and can be taught at any point during instruction.

21st-Century Mathematics Assessment

"An excellent mathematics program ensures that assessment is an integral part of instruction, provides evidence of proficiency with important mathematics content and practices, includes a variety of strategies and data sources, and informs feedback to students, instructional decisions, and program improvement" (NCTM, 2014, p. 5). Teachers who regularly utilize well-designed assessments to adapt instruction methods see students' learning improve (Frey et al., 2018; NRC, 2005; Wiliam & Thompson, 2017). Historically marginalized students have also been disadvantaged by traditional state and national standardized tests; high-quality and highly evolved assessment at the classroom level is essential to accurately determine how all students are progressing across a range of domains, as well as what they need going forward (Feldman, 2019; Jimenez & Modaffari, 2021).

The *California Mathematics Framework* strongly advocates for an equitable, effective assessment system at every level of testing. "As increasingly modern assessments continue to replace traditional tests, all educational assessment should share a common purpose: collecting evidence to enhance student learning and to support students' development of positive mathematics identities.... Measurements of learning that are most helpful are those that assess students' breadth of knowledge and understanding of mathematical content and practices and that require students to reason and solve problems" (CDE, 2024, Ch. 12, p. 2).

HMH Into Math California provides ongoing, balanced assessment and reporting that additionally utilizes digital technologies to empower teachers with data-driven decision-making and tools for effective instructional planning. The program also provides grouping and resource recommendations. This solution yields critical feedback loops that encourage students' self-assessment and reflection while freeing teachers from guesswork and time-consuming assessment reporting and subsequent material selections and planning. These approaches to evaluation of learning support optimal instructional practices and drive positive outcomes for every student.

Formative Assessment

"Formative assessment is a process teachers and students use during instruction that provides feedback to adjust ongoing teaching moves and learning tactics. It is not a tool or an event, nor a bank of test items or performance tasks. Well-supported by research evidence, it improves students' learning in time to achieve intended instructional outcomes" (Linquanti, 2014, p. 2). The *California Mathematics Framework* highlights how ongoing research and evidence on formative assessment illustrate its integral role in improving student learning on a timeline that yields targeted outcomes. "One of the strengths of formative assessment is the flexibility that it affords a classroom teacher, both in timing and approach" (CDE, 2024, Ch. 12, p. 17). The *Framework* points to guidance on effective formative assessment from the Regents of the University of California (2022) which includes clarifying, eliciting, interpreting, and acting—in accordance with a wide body of research presented here.

Formative assessment is a dynamic process that involves gathering evidence of student thinking through formal (e.g., quizzes, assignments) and informal (e.g., discussion, observation) methods. It serves as the bridge between teaching and learning, ensuring that instruction continually adapts to student needs (Greenberg & Walsh, 2012; Hattie & Clarke, 2018; Heritage, 2013; Jimenez & Modaffari, 2021; Tibbitt, 2020; Wiliam, 2018). Effective formative assessment provides immediate, meaningful, and actionable feedback while actively engaging students in monitoring their own progress (Brookhart, 2008; Frey et al., 2018; Hattie & Clarke, 2018; NASEM, 2018; NRC, 2012; Stecker et al., 2005; Wiggins, 2012).

Diagnostic assessments identify students' prior knowledge and skill levels, enabling them to tailor instruction accordingly (Mayes et al., 2008; Ketterlin-Geller & Yovanoff, 2009). Screening tools help pinpoint the students who need additional support (Fuchs & Fuchs, 2006). Ongoing assessment ensures that teaching is responsive to students' individual progress and learning goals (Carpenter et al., 2015; Fonger et al., 2018; Thanheiser & Melhuish, 2023).

Formative assessment is crucial for at-risk students, as it helps close achievement gaps through tailored instruction (Black & Wiliam, 1998a, 1998b; Heritage, 2013; Jimenez & Modaffari, 2021; NCTM, 2016, 2020; Tibbitt, 2020; Xenofontos, 2019). Research supports data-driven mathematics instruction, showing effective formative assessment prevents difficulties and improves learning outcomes for individual students and as well as for the whole classroom (Baker et al., 2002; Clarke & Shinn, 2004; Hattie et al., 2017; Klute et al., 2017; Lee et al., 2020; Stecker et al., 2005; Wiliam, 2018).

Maximizing its benefits requires fostering student agency by explicitly outlining learning goals, success criteria, and feedback, empowering students in self-assessment (Frey et al., 2018; Hattie & Clarke, 2018; Heritage, 2013). Encouraging metacognitive self-reflection enhances achievement (Bond & Ellis, 2013; Desoete & De Craene, 2019; Lee et al., 2020; Schneider & Artelt, 2010). "A central goal of formative assessment is encouragement of students to take responsibility for their learning" (CDE, 2024, Ch. 12, p. 16).

How *Into Math California* Delivers

Into Math California equips educators with a flexible, comprehensive, and accommodative formative assessment toolbox for monitoring student progress through fair and accurate measurement to continually guide and inform instruction.

Effective Formative Assessment that Helps Teachers Clarify, Elicit, Interpret, and Act

Into Math California provides a variety of print and digital assessment options that allow teachers to check student understanding and use both formative and summative data to make instructional decisions. These included module-level prerequisite checks, lesson-level quick checks, module reviews, module assessments, module projects, and beginning-, mid-, and end-of-year assessments, such as *MAP Growth*.

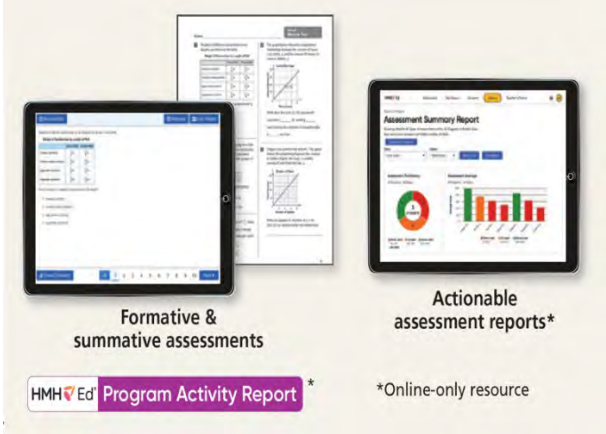
Optimal Formative Assessment to Inform Instruction and Practice

Students take formative assessments before lesson practice, allowing teachers to spend time with small groups correcting misconceptions before they "stick."

Students practice in ways that best support their learning, whether independently, in teacher-led small groups, or in collaborative groups. Teachers also have the choice of lesson practice, leveled practice, or ongoing adaptive practice with every lesson.

Formative Assessment That Yields Data Insights

HMH Classcraft offers embedded, ongoing Quick Checks that, along with Program Assessments, provide data insights to inform instruction.



Summative Assessment

"Education policy and practice must encompass a broader range of assessments so that schools have complete and effective assessment systems. This system would include predictive, informative, and evaluative assessments based on the state's standards and curriculum" (Jimenez & Modaffari, 2021, p. 18). Summative assessment measures overall student achievement at a specific point in time, distinguishing it from formative and diagnostic assessments. It serves as a tool for evaluating student understanding, shaping instruction, and personalizing learning. Summative assessments also function as accountability measures, aligning student performance with standards, and include classroom-based assessments, interim or benchmark tests, and large-scale standardized exams such as the CAASPP or SAT. These assessments typically occur at the end of a unit, quarter, or academic year (CDE, 2024).

Research suggests that well-designed classroom summative assessments can positively impact learning (Harlen, 2005; Moss, 2013; NCTM, 2016). While often linked to high-stakes testing, summative assessments in the classroom can provide valuable insights to inform future instruction (Black & Wiliam, 1998a, 1998b). However, NCTM (2016) cautions against using large-scale assessments as the sole measure for high-stakes decisions about schools, teachers, or students. Instead, a balanced approach that incorporates both formative and summative assessments is recommended, alongside systemic support for high-quality mathematics instruction.

The *California Mathematics Framework*, influenced by work from Yale's Poorvu Center for Teaching and Learning, outlines the best practices for summative assessment (CDE, 2024). These guidelines include using rubrics with clear performance criteria, designing effective and comprehensive questions, and setting clear expectations regarding length, depth, and deadlines. The *Framework* also emphasizes the importance of providing accommodations for students with disabilities.

Performance-based assessments play a crucial role in a well-rounded mathematics assessment system. These tasks, grounded in real-life scenarios, require students to classify, compare, analyze, or evaluate concepts, fostering higher-order thinking (Hibbard, 1996). Such assessments align with classroom learning goals and provide valuable insight into student understanding (Fox, 2004). Performance tasks emphasize the skills and content essential for deep learning and real-world application (Lane, 2013). High-performing education systems globally prioritize performance-based assessments, which emphasize critical thinking, problem-solving, and interdisciplinary knowledge application (Darling-Hammond, 2010).

Research supports the continued use of these assessments despite the pressures of high-stakes testing, as they offer meaningful measures of student competency and promote effective classroom instruction (Schneider et al., 2013). However, assessments should not be merely imposed on students; instead, they should serve as a tool to enhance students' learning (NCTM, 2014).

How *CA Into Math* Delivers

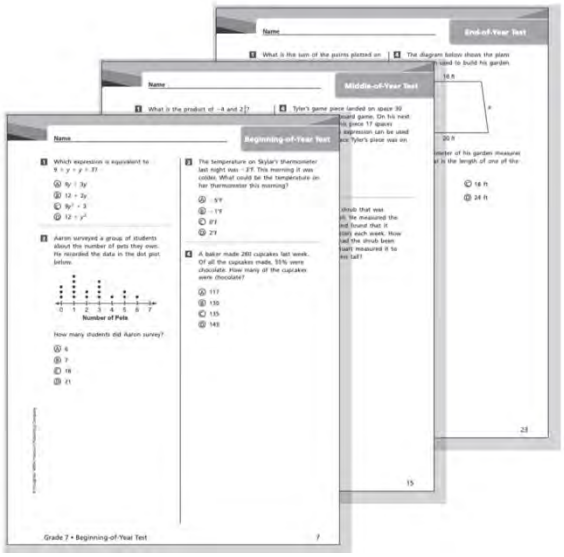
Effective Summative Assessment That Helps Educators Monitor Growth Over Time

In addition to a wide range of flexible formative assessments available in print and digital formats, *Into Math California* provides summative measures and data reporting to allow teachers and administrators to track students' progress and growth over time. The program also includes professional development supports to aid teachers in optimal administration and usage of summative assessment.

Educators can also use *HMH Ed* reports to determine students' proficiency levels and learning needs based on program assessments.

Optimizing Interim Assessment

Into Math California provides a range of interim assessments with reporting that can be administered throughout the school year. These include *MAP Growth* (an optional Connected Resource); the Beginning-of-Year Test, the Middle-of-Year Test, and the End-of-Year Test; as well as assessment(s) supplied by districts. Each assessment functions distinctly and in conjunction to provide specific data, measure student progress, and utilize results in support of diverse learners. These interim assessments should be used at strategic points over the school year to identify and track each student's academic growth and achievements.



Equitable Assessment Practices

Research highlights how assessments shape students' perceptions of their abilities (Aguirre et al., 2013), stressing the need for fair grading practices, such as standards-based grading, reassessment opportunities, and descriptive feedback (Feldman, 2019). Equitable assessments incorporate teacher observations and student portfolios to ensure that all students have multiple, ongoing opportunities to demonstrate their understanding (Jimenez & Modaffari, 2021). The *California Mathematics Framework* advocates for equitable assessment practices to enhance student learning and support positive mathematical identities (CDE, 2024).

Research indicates that formative assessment is particularly crucial for equity. In its review of studies examining formative assessment, the National Mathematics Advisory Panel (2008) concluded that "use of formative assessments benefited students at all ability levels" (p. 46). However, formative assessment is especially beneficial for lower-performing and at-risk students, including those historically underserved due to ethnicity, poverty, and disabilities and those enrolled in special education programs. Research has long shown that continually monitoring student progress, providing feedback, and directly involving students in classroom assessment processes helps shrink achievement gaps and improve overall achievement in mathematics (Black & Wiliam, 1998a & 1998b; DeSilva, 2020; NCTM, 2020; Xenofontoas, 2019). Additionally, per Zimmerman and colleagues (2020), performance-based assessments in STEM subject areas provide an alternative to standardized achievement tests because they enable a more holistic and comprehensive evaluation of a student's knowledge and skills. While this is beneficial for all learners, it is especially helpful to those underrepresented in STEM talent identification.

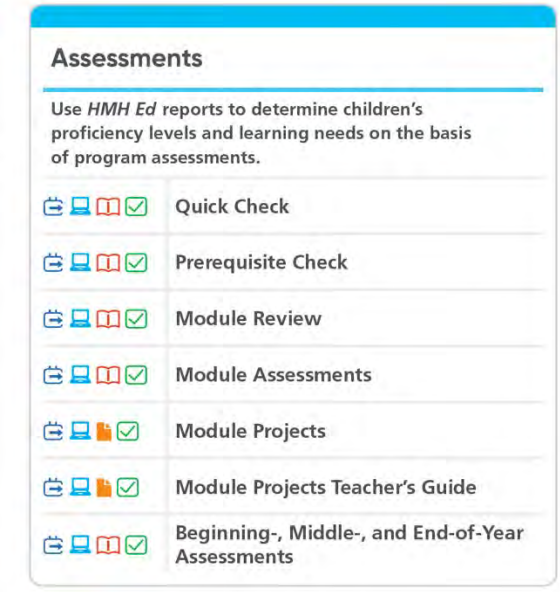
Research associates timed testing with math anxiety and deems it counterproductive, potentially damaging, and disadvantageous for math learning, particularly for students with learning differences. Therefore, the *Framework* emphatically discourages timed testing (Beilock, 2011; Engle, 2002; Henry & Brown, 2008; Ramirez et al., 2013) and, instead, calls for engaging conceptual tasks that develop fluency. Aligned with UDL principles, assessments should allow multiple modes of expression, such as verbal explanations, drawings, and manipulatives, which are particularly beneficial to ELs and marginalized students (Meyer et al., 2014). Additionally, per the *Framework*, mathematics assessments should be culturally responsive and agentic (DeSilva, 2020), while providing actionable feedback (Lambert, 2020).

The *Framework* further recommends the pedagogical approach known as Complex Instruction, which is grounded in sociological research (Cohen et al., 1994; Cohen & Lotan, 2014) and is supportive of the larger aims of the CCSM. Complex Instruction promotes equitable, collaborative learning through positive interdependence, individual accountability, and equal participation. The *Framework* further advises structuring group work to encourage diverse interactions and formative assessment opportunities, fostering optimal learning environments for all students (CDE, 2024).

How *Into Math California* Delivers

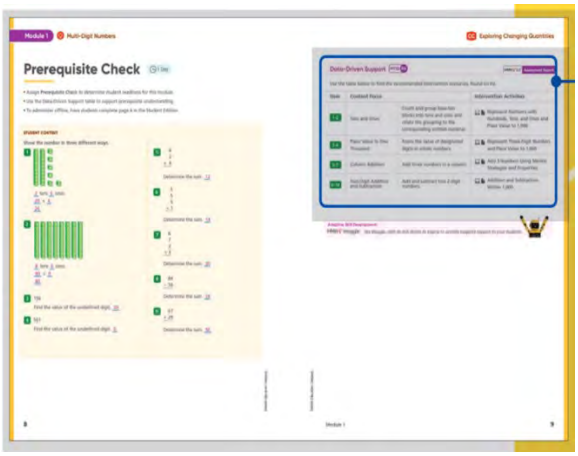
The print and digital assessments within *Into Math California* are available in multiple formats to provide flexibility, equitability, and personalization, meeting the needs of all learners.

Varied and Equitable Assessment Options



Data-Driven Prerequisite Checks Foster Student Success

Each module's Prerequisite Check assesses the prerequisite concepts required to succeed with new learning, providing educators with data that highlight students' strength and areas for improvement. This data guides just-in-time differentiated instruction before module learning begins.



Addressing Misconceptions to Support Accurate Long-Term Learning

Additionally, *Into Math California* formative assessments occur before students practice, so teachers can spend time with small groups correcting misconceptions before they "stick."

Assessment Strategies for English Learners

The California Mathematics Framework (CDE, 2024) aligns with the state's English Language Arts/English Language Development Framework (CDE, 2014) to provide effective assessment strategies for ELs and multilingual students. Key strategies include setting clear learning goals and success criteria, monitoring progress with formative assessments, providing feedback that promotes self-assessment, and fostering a collaborative classroom culture.

Per the *Framework* (CDE, 2024, Ch. 12), assessment for ELs should focus on Student Strengths (content understanding and linguistic assets) and Areas for Growth (skills needing improvement). Consistent with UDL principles, equitable assessment practices should offer multiple means of expression (CAST, 2018; Lambert, 2020). When possible, students should be allowed to demonstrate understanding in their native language. The *Framework* emphasizes that mathematical understanding can be communicated through speech, writing, drawings, and models (CDE, 2024).

Traditional assessments often fail to capture the full capabilities of multilingual learners, as they are designed for monolingual students (De Backer, 2017). Instead, assessments should leverage students' full linguistic repertoires. Formative assessment, which provides ongoing feedback to teachers and students, is crucial for EL success (CDE, 2014). Teachers should assess students' mathematical reasoning and language use over time (CABOE, 2023).

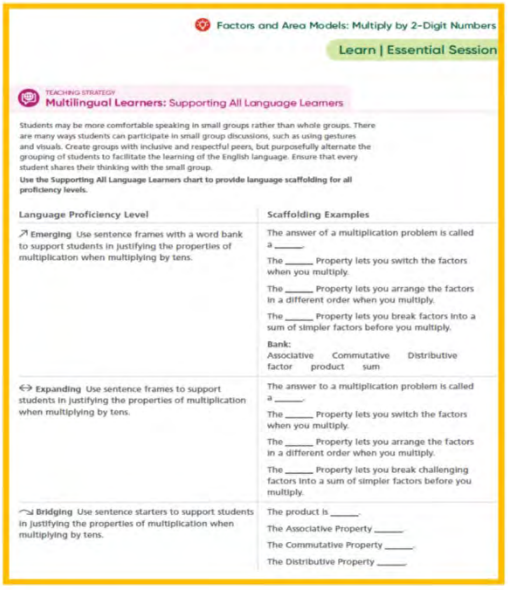
The *Framework* cites examples of assessments reflective of best practices for ELs. The California Assessment of Student Performance and Progress (CAASPP) incorporates multifaceted assessments of reasoning and problem-solving and includes a Spanish-language version (CADOE, 2023). The Smarter Balanced CAASPP assessment is available in Spanish in a "stacked version" that presents questions in both languages. Language development strategies such as "Collect and Display" (Zwiers et al., 2017) help teachers track students' mathematical vocabulary growth. Providing rubrics with key vocabulary and structured opportunities for self- and peer assessments enhances clarity.

The *Framework* further suggests assessment modifications for ELs, including verbal responses, multiple response formats, bilingual dictionaries, chunking assessments, and group assessments. Qualified bilingual professionals should assist in ensuring accessibility. Effective feedback highlights strengths, clarifies misunderstandings, and allows students to revise their work. For summative assessments, teachers should analyze language demands, provide opportunities for students to practice mathematical reasoning, and offer feedback before testing. These practices support ELs in demonstrating their full mathematical abilities. (CDE, 2024).

How *Into Math California* Delivers

Ongoing, Formative, Flexible Assessment for English Learners That Aligns With Best Practices

Into Math California offers multiple means for assessing and tracking students' language development and progress toward the ELD standards. In the Quick Check for each lesson, the Teacher's Guide describes a formative assessment that can be used to check students' mastery of the lesson's Language Objective. Teachers can also use the MLRs and Turn & Talks of each lesson to observe and chart the growth of each student's academic language. Furthermore, the observational rubric found in the Language Development Resource Guide provides a ready means for documenting students' academic language growth. This documentation can help teachers and students as well as students' families track student growth over time.


Factors and Area Models: Multiply by 2-Digit Numbers

Learn | Essential Session

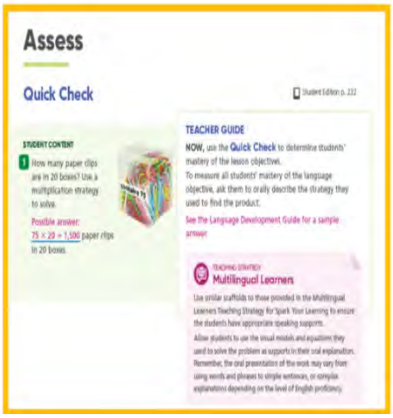
TEACHING STRATEGY
Multilingual Learners: Supporting All Language Learners

Students may be more comfortable speaking in small groups rather than whole groups. There are many ways students can participate in small group discussions, such as using gestures and visuals. Create groups with inclusive and respectful peers, but purposefully alternate the grouping of students to facilitate the learning of the English language. Ensure that every student shares their thinking with the small group.

Use the Supporting All Language Learners chart to provide language scaffolding for all proficiency levels.

Language Proficiency Level	Scaffolding Examples
↗ **Emerging** Use sentence frames with a word bank to support students in justifying the properties of multiplication when multiplying by tens.	The answer of a multiplication problem is called a _____. The _____ Property lets you switch the factors when you multiply. The _____ Property lets you arrange the factors in a different order when you multiply. The _____ Property lets you break factors into a sum of simpler factors before you multiply. **Bank:** Associative Commutative Distributive factor product sum
↔ **Expanding** Use sentence frames to support students in justifying the properties of multiplication when multiplying by tens.	The answer to a multiplication problem is called a _____. The _____ Property lets you switch the factors when you multiply. The _____ Property lets you arrange the factors in a different order when you multiply. The _____ Property lets you break challenging factors into a sum of simpler factors before you multiply.
↝ **Bridging** Use sentence starters to support students in justifying the properties of multiplication when multiplying by tens.	The product is _____. The Associative Property _____. The Commutative Property _____. The Distributive Property _____.

These learning activities provide ongoing opportunities for effective, evidence-based formative assessment that can be used with confidence to guide instructional decision-making and monitor language proficiency development levels (Emerging, Expanding, or Bridging) for each student. The program also includes ongoing support for educators in how to best implement these assessment strategies.


Assess

Quick Check

STUDENT CONTENT

1 How many paper clips are in 20 boxes? Use a multiplication strategy to solve.

Possible answer:
75 × 20 = 1,500 paper clips in 20 boxes.

TEACHER GUIDE

NOW, use the **Quick Check** to determine students' mastery of the lesson objectives.

To measure all students' mastery of the language objective, ask them to orally describe the strategy they used to find the product.

See the Language Development Guide for a sample answer.

TEACHING STRATEGY
Multilingual Learners

Use similar scaffolds to those provided in the Multilingual Learners Teaching Strategy for Spark Your Learning to ensure the students have appropriate speaking supports.

Allow students to use the visual models and equations they used to solve the problem as supports in their oral explanation. Remember, the oral presentation of the work may vary from using words and phrases to simple sentences, or complex explanations depending on the level of English proficiency.

Formative Assessment

The Quick Check contains a task that teachers can use to measure and assess students' language development throughout the lesson. This formative assessment is designed to look at the development of the language skill, ELD standard, and mathematical content of the lesson in one comprehensive approach. Sample answers to the Quick Check language question can be found in the Language Development Resource Guide.

Using Evidence to Inform Learning, Grouping, and Differentiation

The *California Mathematics Framework* (CDE, 2024) emphasizes the importance of continuous, data-driven assessment to support responsive and equitable mathematics instruction. Collecting and analyzing assessment data at regular intervals from a broad range of standards-aligned sources to inform instruction and undergird support for individual students is essential to responsive, equitable approaches to teaching (Jimenez & Modaffari, 2021; NCTM, 2020).

Internationally, high-performing education systems prioritize formative, flexible, and equitable assessments. Data-driven instruction involves the structured use of both qualitative and quantitative assessments to optimize learning experiences and ensure all students meet learning goals (Darling-Hammond, 2010; Greenberg & Walsh, 2012; Wiliam & Thompson, 2017). Comprehensive assessment systems support progress monitoring, performance evaluation, feedback, and curriculum appraisal (Wohlstetter et al., 2008). Research demonstrates that well-implemented data-driven instruction can significantly enhance student achievement (Bambrick-Santoyo, 2014; Dunn et al., 2013; Hamilton et al., 2009; Mandinach & Schildkamp, 2021; Marsh et al., 2006; Schifter et al., 2014). Successful implementation requires the formative use of data to adapt instruction and learning experiences based on assessment insights (Hoogland et al., 2016). Emerging and advanced technologies enhance access to multiple data sources, improving instructional decision-making (Mandinach & Schildkamp, 2021).

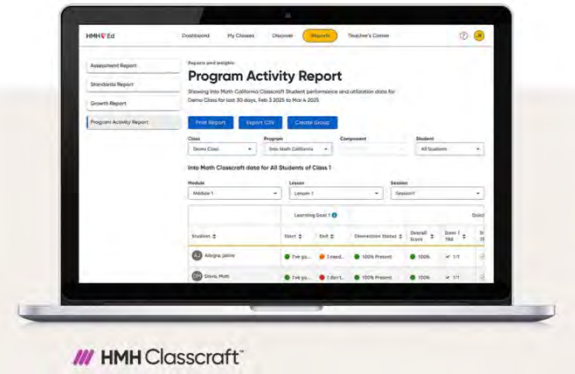
Differentiated instruction is a well-established, evidence-based, equity-driven practice of flexibly altering teaching that maximizes learning for all students and yields positive outcomes across achievement levels, including specifically within mathematics. A differentiated approach to instruction recognizes and supports the classroom as an inclusive community where students are nourished as individual learners and provided with an appropriate, motivating balance of challenge and success. In effective differentiated environments, instruction is responsive to students' varying readiness, interests, and learning profiles. This method offers students varying levels of assessment-based expectations for task completion within a lesson or unit (Chamberlin & Powers, 2010; Kalbfleisch & Tomlinson, 1998; Lawrence-Brown, 2004; NMAP, 2008; Rogers & Garii, 2013; Tomlinson, 1997, 2001, 2005, & 2017). "When teachers differentiate how they support student learning, they are ensuring that all students have access to the content and skills that have been identified in the standards and that educational opportunities are equitable for all students" (Rogers & Garii, 2013, pp. 34–35). The *Framework* highlights differentiation as key to supporting all students, ensuring equitable access to rigorous mathematics education (CDE, 2024). It discourages fixed-ability tracking and instead advocates for scaffolding, UDL principles, and flexible grouping to foster an inclusive, collaborative environment focused on problem-solving. By leveraging formative assessments and comprehensive data systems, educators can make informed decisions to support students' diverse needs while maintaining high expectations for all learners.

How *Into Math California* Delivers

Into Math California was designed to support differentiation that most effectively meets the evolving needs of every student. The program includes abundant practice as well as three small-group activities for each lesson. Teachers can choose the best practice and/or activity according to their classroom observations and the formative data collected during whole-class instruction.

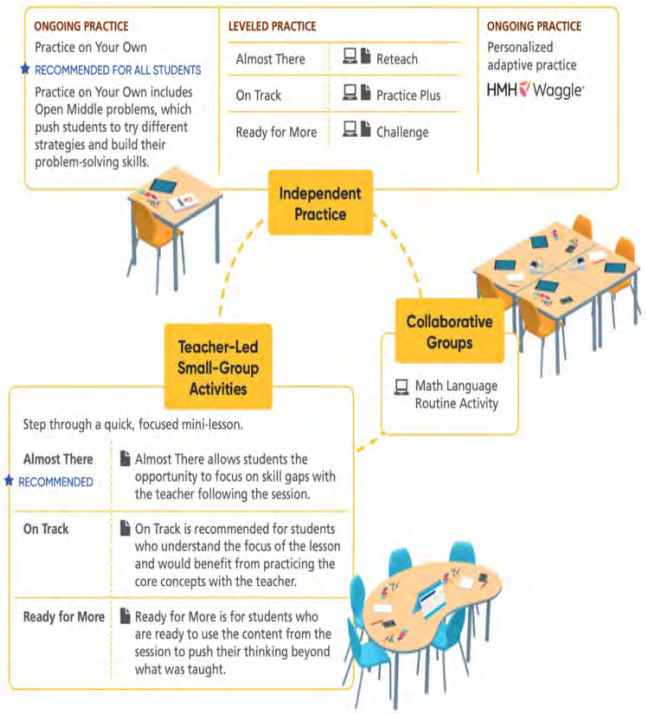
Rely on Real-Time Data

Teachers use real-time formative assessment data from the *HMH Classcraft* Quick Check on learning goals to make instructional decisions and group students on the spot.



Group and Assign Practice for Optimal Differentiated Learning

Educators are empowered to make data-driven decisions about which students will benefit most from the teacher-led small-group activities and can work with additional groups as time allows. For students not working in groups, teachers can choose the most beneficial practice for that individual, which is provided in *HMH Waggle*.



Strategic Technology Use to Improve Mathematics Learning

"An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking" (NCTM, 2014, p. 78). Over the past decade, policies and practices regarding technology use in classrooms around the country have shifted incrementally to widespread—and widely varying—application. Concurrent with such trends, there has been an emergence of growing evidence attesting to the positive impacts of technology in education as technology itself has advanced through innovation. No longer a question of whether technology can improve learning, the issues evolved as experts explored how technology could deliver improved learning outcomes for all students. Digital learning has enormous potential to positively transform education for diverse groups of students when evidence-based practices are incorporated into instructional design. Advances in educational technology provide tremendous opportunities to promote growth for all students by providing engaging, evidence-based content; increasing student agency over pace, environment, and interests; and enabling more frequent, timely feedback. These tools also help advance educational equity by expanding access to high-quality learning experiences for historically disadvantaged students (Chen et al., 2018; Curtis & Werth, 2015; Hattie, 2009, 2012; 2023; Johnson et al., 2023; Martin et al., 2020; Patrick & Powell, 2009). Since the start of the 21st century, educators in the United States have broadly adopted the understanding that “[t]echnology can be a powerful tool for transforming learning. It can help affirm and advance relationships between educators and students, reinvent our approaches to learning and collaboration, shrink long-standing equity and accessibility gaps, and adapt learning experiences to meet the needs of all learners” (U.S. Department of Education [USDOE], 2017, p. 3).

The *California Math Framework* emphasizes that effective integration of technology can support rigorous curricula, equitable access, and inquiry-based instruction. It encourages educators to leverage technological tools to create engaging and interactive learning experiences, thereby fostering deeper understanding of mathematical concepts. Additionally, the *Framework* acknowledges the importance of preparing students for a technology-driven world by incorporating digital tools that reflect real-world applications of mathematics (CDE, 2024).

HMH Into Math California harnesses technology to provide dynamic, interactive, adaptive, and personalized instruction along with practice and assessment solutions addressing individual students' ongoing needs and engaging their interest in math learning.

Developing Meaningful, Effective Digital Learning Environments

The *California Mathematics Framework* underscores the vital role of technology in fostering meaningful mathematics learning through digital platforms. It emphasizes student-centered learning and aligns with the California Common Core State Standards for Mathematics (CA-CCSSM) while promoting conceptual understanding, procedural fluency, and equitable access for diverse learners (CDE, 2024). *The Digital Learning Integration and Standards Guide: Mathematics* (DLI&SG) provides research-based practices to support effective technology use, including fostering collaboration, active learning, and student agency (CDE, 2021).

Strategic technology integration, as endorsed by the National Council of Teachers of Mathematics (NCTM, 2015b), enhances mathematics instruction when used purposefully rather than continuously. Teachers should prioritize mathematical learning objectives while leveraging the benefits of technology (Dick & Hollebrands, 2011; NCTM, 2015b). Effective technology use should avoid a "technocentrist" approach, where the focus is on technology itself rather than its pedagogical value (Zinger et al., 2017). Two primary types of educational technologies—content-specific tools that support conceptual exploration and content-neutral tools that aid problem-solving and communication—should be used strategically to develop students' mathematical reasoning (NCTM, 2015b).

The USDOE (2019) highlights technology's critical role in STEM education, enhancing learning experiences through dynamic representations, simulations, interactive models, and real-time feedback. Digital platforms also facilitate formative and summative assessments, supporting interdisciplinary and project-based learning.

The *Framework* references Koehler and Mishra's (2009) TPACK model, which outlines essential educator competencies for effectively integrating technology. Teachers should leverage their knowledge of student learning styles and cognitive processes to design technology-enhanced experiences that encourage creativity, conceptual understanding, discourse, and individualized learning strategies. Additionally, digital tools can support student-centered learning by fostering reflective practices and metacognition.

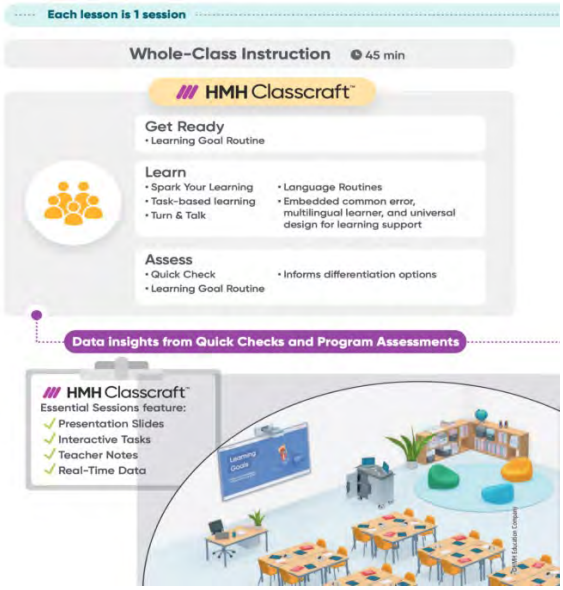
Assessment integration is another critical component, as technology allows students to engage with a variety of assessment formats, such as digital portfolios and recorded reflections, which align with the California Assessment for Student Progress and Performance (CAASPP) (CDE, 2024). Within both in-person and remote learning environments, formative assessment remains a powerful tool for tracking progress and driving instructional decisions.

In sum, the *Framework* advocates for strategic and meaningful technology use that enhances mathematical learning, supports equity, and prepares students for success in a technology-driven world.

How *Into Math California* Delivers

Technology-Enhanced, Optimized Math Learning Experiences

Into Math California classroom environments are enriched and empowered with *HMH Classcraft*. This digital learning solution increases students' engagement and interactivity with math content while providing teachers with a broad range of effective instructional planning and classroom-management tools.



Additionally, *HMH Classcraft* yields real-time data from ongoing, varied, and embedded formative assessment measures to guide differentiated instruction at optimal challenge levels, ensuring all students meet California's rigorous mathematics standards.

Interactive Experiences and Tools to Enrich the Learning of Math Content and Practices

"Technology use in the teaching of mathematics has one primary purpose: to facilitate interactive experiences that enrich the learning of both content standards and SMPs" (CDE, 2024, Ch. 11, p. 2). Research highlights the benefits of blended learning, which increases student engagement and motivation while providing personalized instruction (Anthony, 2019; Halverson & Graham, 2019; Kwon et al., 2019; Moore et al., 2017; O'Byrne & Paytash, 2015; Patrick & Sturgis, 2015). Blended learning fosters student success by offering flexibility, autonomy, and timely feedback, particularly benefiting historically disadvantaged students through greater access to high-quality education (Curtis & Werth, 2015; Johnson et al., 2023; SRI International, 2018; USDOE, 2017).

Digital learning tools promote engagement by integrating multimedia, interactivity, and student agency. Studies show that students learn better when verbal instruction is supplemented with visual elements (Johnson et al., 2023; Mayer, 2013, 2017). Multimedia and digital tools create engaging environments, increasing motivation and persistence for students facing challenging tasks (Chen et al., 2018; Taylor & Parsons, 2011). Student autonomy in digital learning—such as encouraging students to make meaningful choices—has been linked to improved motivation and resilience (Ryan & Deci, 2000; NASEM, 2018). Furthermore, online collaboration tools strengthen classroom communities and communication skills (SRI International, 2018; Wolfe & Poon, 2015).

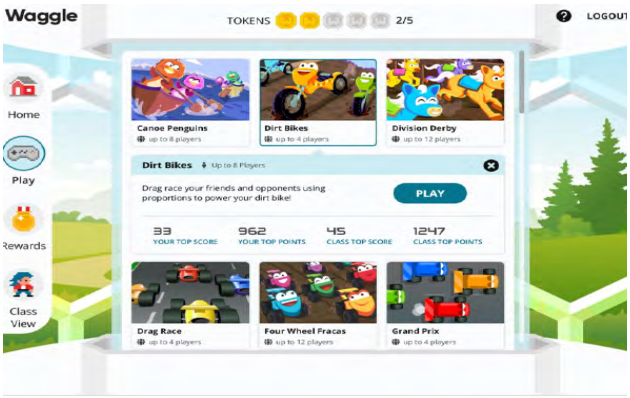
Study results indicate that technological tools can enhance formative assessment by providing timely feedback, progress monitoring, and individualized instruction (Anthony, 2019; Hamilton, 2018; Johnson et al., 2023; Lee et al., 2020), particularly in mathematics education (Faber et al., 2017; Haelermans, & Ghysels, 2013) and for learners with special needs (Alhadi et al., 2023). Adaptive learning technologies offer personalized feedback to benefit all students (Alhadi et al., 2023; Faber et al., 2017). When instructional goals are clear, digital tools help students demonstrate learning effectively (Hattie, 2023; Hamilton, 2018). Further, by providing a diverse array of online and other digital resources, technology supports learning drawn from real-world challenges and students' personal interests and passions while also aiding the organization of a project-based curriculum, all of which are especially salient and effective within STEM instruction (USDOE, 2017, 2019).

For educators, technology simplifies tasks such as data management and assessment scoring, freeing time for personalized instruction and small-group learning (Moore et al., 2017; Wolfe & Poon, 2015). Automated scoring tools aid teachers in evaluating student reasoning and providing adaptive guidance (Gerard et al., 2015). Ultimately, digital tools help educators "take advantage of the things that leading brick-and-mortar schools do well, such as creating a strong, supportive culture that promotes rigor and high expectations for all students, as well as providing healthy, supportive relationships and mentorship" (Horn & Staker, 2011, p. 7).

How *Into Math California* Delivers

Personalized Adaptive Learning That Propels Students Toward Success

Into Math California provides personalized, adaptive instruction and practice supported by *HMH Waggle*.



Teachers can assign content directly aligned to daily instruction, or they can utilize the auto-assignment feature to ensure students receive targeted practice. Powered by a skill-based AI engine, the adaptive practice adjusts in real time to meet each student's needs and provides powerful data insights to inform instruction.

Professional learning

To support the delivery of effective instruction, *Into Math California* features research-based approaches to professional learning that prepares teachers to facilitate high-impact learning experiences for their students. Comprehensive professional learning solutions are data and evidence driven, mapped to instructional goals, and centered on students, and they build educators' collective capacity. HMH enables teachers to achieve agency in their professional growth through effective instructional strategies, embedded teacher support, and ongoing professional learning relevant to everyday teaching. In addition, school leaders can help support teachers' *Into Math California* implementation through leader-facing reports, including reviewing teachers' participation and completion of *Teacher Success Pathways* and their teacher-coach engagement.

This chapter discusses *Into Math California's* professional learning offerings and how these align to three key areas of research on effective professional learning, including connected and personalized professional learning, program-aligned professional learning for actionable insights, and coaching to strengthen teaching and learning.

Connected and Personalized Professional Learning

Effective curriculum-based professional learning involves ongoing, active experiences that enhance the rigor and impact of instructional practices and ideally replicate the learner-centered approaches teachers are expected to provide for their students. Key elements include high-quality, educative curriculum materials; transformative learning experiences that shift teachers' attitudes, beliefs, and practices; and a strong emphasis on equity to ensure all students meet high expectations. Functional design elements include learning designs that model inquiry-based instruction, experiences to shift teachers' beliefs, opportunities for reflection and feedback, and change-management strategies that address individual concerns and group challenges. Finally, structural design features include collective participation, where teachers practice and reflect on the curriculum; progressive models of learning that move from initial implementation to ongoing support and capacity building; and the efficient use of teachers' time. These elements of effective curriculum-based professional learning must operate within in a system characterized by strong leadership, adequate resources, and alignment toward common goals (Short & Hirsh, 2020).

How professional learning is delivered has an impact on its effectiveness. Professional learning programs with teacher-to-teacher collaboration focused on instructional improvement—whether in professional learning communities (PLCs), teacher teams, or group work in professional learning sessions—have demonstrated improvement in teachers' instructional skills. Another effective practice is conducting follow-up meetings or coaching sessions after the initial implementation of a program so that teachers can share their experiences and receive feedback. The content of the professional learning is equally important; it should focus on subject-specific instructional practices (not merely content knowledge), prioritize specific supportive materials over general principles, and help teachers build stronger relationships with students (Hill & Papay, 2022).

Long-term connected professional learning includes cohesive features, such as online coaching, observations, and collaboration, while focusing on student well-being and meaningful student learning in digital environments. A connection between workshops, coaching, and collaboration is essential for a professional learning program to positively impact student achievement. Connecting workshops, follow-up coaching, and support among peers can help teachers retain new information, apply and refine new skills, and share effective and scalable approaches (Aguilar, 2019).

Effective professional learning, whether in-person, online, or blended, offers teachers coherent experiences that directly connect their classroom practice and build instructional proficiency. This approach includes alignment between the study of theory and practice, observation of theory and practice, individual coaching, and further practice and refinement through collaboration. Each of these components is essential for supporting and building on the content and pedagogy that is learned, observed, and practiced in each of the other components (Rock, 2019).

For schools to successfully implement high-quality instructional materials, effective professional learning during the curriculum launch—when teachers are learning and committing to an instructional approach—is critical (Gulamhussein, 2013). Teachers' initial exposure to a concept should engage them

through varied approaches and active learning strategies to make sense of the new practice (Bill & Melinda Gates Foundation, 2014; Garet et al., 2001; Gulamhussein, 2013). An effective professional learning program should be curriculum-based and focused on targeted content, strategies, and practices (Bill & Melinda Gates Foundation; 2014; Saxe et al., 2001) and be grounded in the teacher's grade level or discipline (Gulamhussein, 2013).

Online professional learning can help address resource challenges in implementing a scalable and sustainable instructional model. Such platforms can create a peer-to-peer support community, building the teaching team's capacity to support one another. Perhaps most importantly, online professional learning allows teachers to experience the same agency and personalized learning they are creating for students. The unique advantage of blended professional learning is the shift from learning as a one-time or periodic event to seeing it as an ongoing, embedded practice (Tucker & Wycoff, 2019).

Many school districts and providers of teachers' professional learning are moving toward a more personalized model of professional learning, taking a cue from the movement toward personalized learning for students. This approach often focuses on short modules, which teachers can choose and complete on their own time. The modules can incorporate aspects of gamification, micro-credentialing, and online professional learning communities. By allowing teachers to select their own professional learning courses and activities, as well as complete and revisit content at their own pace, the learning will be better aligned with their needs. Teachers will be able to set goals, find resources to help them meet those goals, refresh their knowledge, track their progress, and get feedback from supervisors and colleagues (Gamrat et al., 2014; Meeuwse & Mason, 2018).

Providing teachers with time and frameworks to collaborate on improving their instruction through PLCs or Teacher Study Groups (TSGs) has the potential to improve teachers' knowledge, instructional practices, and student achievement. (Gersten et al., 2010).

How *Into Math California* Delivers

Into Math California provides a continuum of connected professional learning designed to foster teacher agency, promote collaboration, and build collective efficacy and capacity to support teachers' roles as both facilitators and implementors of high-quality instruction. Through strategic planning, guided implementation support, and blended coaching, HMH helps schools and districts achieve measurable gains with professional learning centered on research-based practices and student outcomes.

HMH Into Math California Implementation Support

Guided Implementation Support for *Into Math California* assists educators in building confidence and success with their *Into Math California* program. A Getting Started session provides teachers with learner-centered foundational program knowledge through exploration and hands-on activities, enhancing their instructional skills and building confidence for successful implementation. Subsequently, a Follow-Up session focuses on using data and *Into Math California* resources to differentiate instruction, empowering teachers to effectively meet the diverse needs of their students and deepen their classroom impact. In addition, a Leader Success session is included to provide school and district leaders with essential program details and resources to support successful implementation.

Ongoing training and support resources are also provided through the *Into Math California Teacher Success Pathway* on *HMH Ed*, HMH's learning platform. There, teachers can access a guided learning pathway focused on the first 30 to 60 days of instruction. A recommended sequence of topics, which includes live events, videos, interactive media, and related resources, helps teachers plan, teach, and assess student learning using their new HMH program. The pathways allow teachers to personalize instruction by selecting the mode of learning (live event during back-to-school or on demand) that best meets their needs. In addition, teachers have control over how much time they spend reviewing resources.

Teacher's Corner

HMH continues to provide a personalized model for professional learning, engaging teachers throughout the school year via *Teacher's Corner* on *HMH Ed.* Teachers have access to a searchable library that offers classroom videos, tips from other teachers, teacher-to-teacher collaboration through live events, and additional content and support from HMH's experienced instructional coaches. *Teacher's Corner* includes the following:

Live events promote learning from HMH coaches, thought leaders, and *Into Math California* educators in live online sessions with active participation and feedback.

Program support provides on-demand teaching resources and professional learning tools, including model lesson videos, teacher tips, interactive support, and more.

The Breakroom inspires educators with ideas from other educators, new lesson resources, and reflection opportunities.

The Leader's Corner offers leader access to program support and resources to assist teachers with program implementation (available to district and site-based leaders through their *HMH Ed* login).

The Family Room provides a dedicated space with personalized, easily accessible, and on-demand resources to support family engagement and learning at home (available to families through their student's *Into Math California* login on *HMH Ed*).

Program-Aligned Professional Learning for Actionable Insights

Grounded in research principles of effective professional learning, HMH program-aligned courses provide flexible, continuous, and relevant learning for teacher groups. Through a blend of in-person and online study group sessions that may be scheduled anytime, HMH courses provide educators with deep learning that ties theory to practice, which is proven to generate sustained impact (Tucker & Wycoff, 2019). HMH courses are designed to unpack the program's research, helping educators understand the *why* behind the program's evidence-based instructional approach and content.

Educators will benefit from opportunities to continuously review their learning, practice implementing strategies in the classroom, and exchange feedback through online study groups. Engaging in follow-up online study groups to practice and reflect after each session reinforces educators' growth through sustained, collaborative learning (Aguilar, 2019). HMH courses do more than share subject knowledge; they offer personalized content aligned with research-based practices, leveraging program resources, activities, and instructional strategies. This approach ensures professional learning is relevant and applicable to classroom practices (Hill & Papay, 2022).

HMH Program-Aligned Courses

The course, *From Reasoning to Action: Building Strong Math Thinkers With* Into Math California, includes three in-person sessions and six online study groups. Participants will explore best practices in mathematical reasoning and discourse to improve student success.

These sessions will help link students' proficiency to understanding and communication, teach them to create a supportive math community, promote discourse and mathematical language, and help them use tools to enhance reasoning.

From Reasoning to Action: Building Strong Math Thinkers with Into Math California

This course provides teachers with immediately applicable learning that uses their program resources to connect pedagogy to practice.

Session 1: Conversations that Count: Fostering Reasoning and Discourse

Session 2: Talk Like a Mathematician: Using Language Routines for Deep Reasoning

Session 3: Make Math Thinking Explicit: Using Tools and Representations

Coaching to Strengthen Teaching and Learning

Research has demonstrated that sustained, job-embedded coaching is the most effective form of professional learning, whether it is delivered in person or online. Coaching delivered in person is most effective when coaches are highly expert and focus their work with teachers on a clearly specified instructional model or program. Other opportunities for teachers to develop their knowledge of the targeted instructional model (e.g., in courses, workshops or coach-led learning groups) are also an important component of successful coaching programs. Online coaching shows promise for being at least as effective as in-person coaching for improving outcomes, though the research base comparing delivery systems is thin. The balance of evidence to date, however, suggests that the medium through which coaching is delivered is less important than the quality and substance of the learning opportunities provided to teachers (Matsumara et al., 2019).

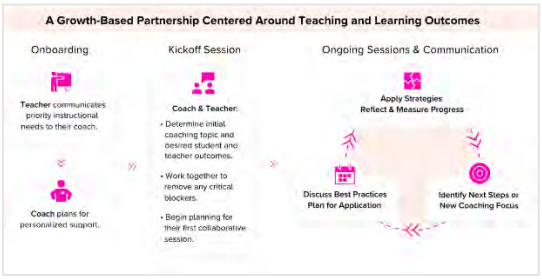
A recent meta-analysis of coaching programs found overall effect sizes of 0.49 SD on instructional practices and 0.18 SD on student achievement. Encouragingly, teachers who received online coaching performed similarly to teachers who received in-person coaching in terms of improving both instructional practices and student achievement. The authors identified several aspects of online coaching as potential strengths: increasing the number of teachers with whom a high-quality coach can work, reducing educators' concern about being evaluated by their coach, and lowering costs while increasing scalability (Kraft et al., 2018).

The best evidence for coaching is found for one-to-one coaching, where coaches observe and offer feedback on teachers' practice. There is not a great deal of research on specific coaching practices, but effective coaches might engage in co-planning, modeling, or guiding teacher reflection. Effective coaching is time-intensive, and the most successful programs have invested in the selection, training, and ongoing support of their coaches (Hill & Papay, 2022).

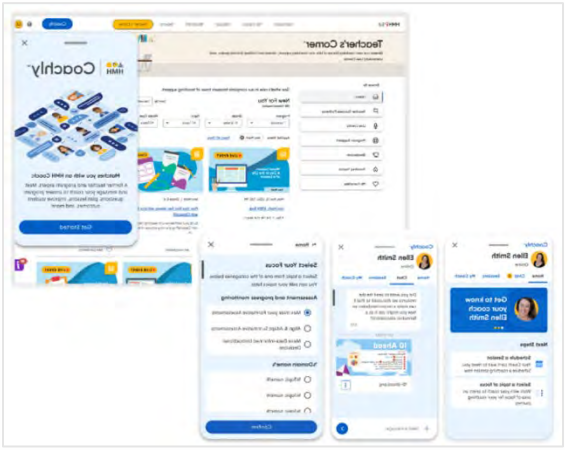
Online coaching can provide a framework for a shared leadership structure that focuses on facilitating teachers' autonomy, self-management, empowerment, and cooperation. Because the coach and the teacher jointly pursue the goal of increased student achievement, virtual coaching provides social support for both parties, leading to enhanced emotional and psychological strength. Any coaching relationship, whether traditional or virtual, builds on several underlying qualities of the teacher and the coach. Chief among these qualities is a willingness to change, a trusting relationship, a high level of initiative, and a personal and organizational commitment to the workplace (Blackman, 2010).

How *HMH Into Math California* Delivers

HMH Coaching is grounded in a research-based coaching framework that leverages instructional best practices that are proven to impact student success. It offers sustained, data-driven, and personalized support aligned to each teacher's individual learning goals. Available through unlimited one-on-one virtual coaching sessions and in-person coaching days, *HMH Coaching* supports every teacher to elevate instructional practice, meet district goals, and raise student achievement. *HMH Coaching* provides teachers with the agency and insights to connect their own growth to student outcomes



HMH Coachly is a yearlong digital coaching subscription that gives teachers unlimited access to a dedicated HMH coach. Teachers are matched with a highly experienced instructional coach who will guide them through high-impact coaching topics such as implementing instructional best practices with *Into Math California*, addressing classroom challenges, and establishing and tracking goals. Once logged onto *HMH Ed*, teachers can schedule unlimited one-on-one virtual coaching sessions, message their coach, and receive timely feedback in a single-platform experience. For additional support, Coachly licenses can be paired with in-person group coaching days.



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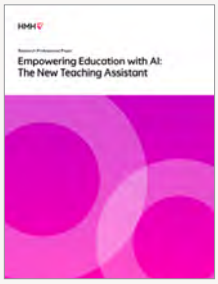

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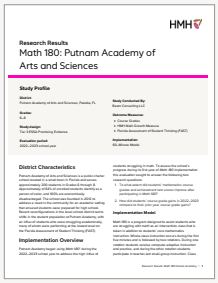

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